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Fundamental Approaches to Legal Logic

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1 Introduction

Since 1950,[1] several jurists have attempted to develop a legal logic and they have made some progress to that end ; however, up to now, legal logic has not been an issue, neither in legal practice nor in legal science, nor legal education. There are several reason for this. For one, it may be mentioned that legal logic has thus far, with few exceptions, aided neither in the analysis of legal problems related to legal theory nor legal practice. In my opinion, the main reason for this is that papers contributing to the development of legal logic have been chiefly concerned

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* This is the revised English version of my German paper: "Zu Ansätzen der Juristischen Logik", in: Tammelo, I. and Schreiner, H. (eds.): *Strukturierungen und Entscheidungen im Rechtsdenken*, Wien-New York (Springer-Verlag) 1978, pp. 277 ff. The German paper was written as my first work as a visiting professor at the Institute for Legal Philosophy and Legal Information Science, University of Munich (Professor DDr. Arthur Kaufmann), which was supported by the Alexander von Humboldt-Stiftung. I am indebted to Professor Kaufmann, my host at that time, who was always supportive. In 1996, I had a opportunity to study at the same institute, also as a Humboldt-fellow, where I worked on this English version. I thank my host, Professor Lothar Philipps, Mr. Helmut Prendinger, who helped me with the translation and Mr. Frank Höfinger, my research assistant for their help and support.

1 U. Klug, *Juristische Logik*, Berlin - Heidelberg - New York 1950, third extended edition 1966; an early work of I. Tammelo, *Drei rechtsphilosophische Aufsätze*, Heidelberg 1948, should be mentioned, too.

with specific logics, e.g., the "logic of norms" or "deontic logic." Although these systems of logic are interesting from a philosophical point of view, they are neither soundly stated nor useful.[2] Moreover, there is a risk of getting distracted by quasi-problems, as will be shown in Section 3 below.

Concerning the method of a legal logic, one need not necessarily develop and apply a special logic of norms; for classical (mathematical) logic, which is almost perfect and has been developing since 1879,[3] is directly applicable to legal norms. As opposed to the logic of norms, the methods of mathematical logic are sound, precisely stated and effective. For this reason, I would like to suggest that jurists no longer be concerned with the special logic of norms. Instead, they should make efforts to directly apply mathematical logic to law.

Hence, in this paper I will advance approaches to legal logic which: (1) concern the direct applicability of mathematical logic to law; (2) concern the problem of paradoxes arising when the logic of norms is applied to legal norms. This will turn out to be only a quasi-problem due to defects in the logic of norms; (3) concern the provability, by means of mathematical logic, of the process of law application as an example of the efficacy of logic in the analysis of legal reasoning.

2 Direct Applicability of Classical Mathematical Logic in Law

Advocates of legal logic who are concerned with the development of a logic of norms argue that legal norms possess properties that are differ-

2 Another problematic point is that the various deduction systems proposed by representatives of logics of norms are more or less disproved, especially with respect to paradoxes. If methods which should be used in the actual analysis of legal problems are always disproved, they cannot be safely and effectively applied.

3 G. Frege, *Begriffsschrift*, Halle 1879.

ent from statements: unlike statements, it is not reasonable to speak of truth and falsity where legal norms are concerned; hence the method of mathematical logic, which is based on the logical truth-values "true" or "false", are not directly applicable to legal norms.[4] Tammelo and Rödig criticized that claim. According to Tammelo, truth-values are extensible, dependent on the context of application, since "true and false in logic are used in a specific logical sense."[5] The truth-values of logic are to be distinguished from the notion of truth in an epistemological sense, such that the problem of the logical notion of truth can be distinguished from the verifiability of a sentence.[6] Following Rödig, truth-values in logic have to be conceived of as "relative." [7] "From a logical point of view, the question is whether, under the assumption that certain objects have certain properties, further objects can be attributed certain properties on formal— 'logical'—grounds." [8]

In this context, it is possible to refer to Tarski's formal concept of truth.[9] Weinberger in his response[10] to Rödig's criticism of his work[11] did not always correctly grasp the meaning of the direct applicability of

4 See, for instance: O. Weinberger, *Rechtslogik*, Wien – New York 1970, p.191.

5 I. Tammelo, *Outlines of Modern Legal Logic*, Wiesbaden 1969, p. 87; cf. also to: I. Tammelo, review of "Heinz Wagner / Karl Haag, Die moderne Logik in der Rechtswissenschaft", in: *ARSP* 58 (1972), p. 448.

6 Cf. J. Rödig, "Über die Notwendigkeit einer besonderen Logik der Normen", in: *Jahrbuch für Rechtssoziologie und Rechtstheorie* (H. Albert, N. Luhmann, W. Maihofer, O. Weinberger, editors), Vol. II (1972), p. 170.

7 J. Rödig, "Logik und Rechtswissenschaft", in: *Rechtswissenschaft und Nachbarwissenschaften 2* (D. Grimm, ed.), München 1976, p. 61.

8 op.cit. (Fn.6), p. 171.

9 A. Tarski, "Der Wahrheitsbegriff in den formalisierten Sprachen", in *Studia Philosophica I*, Leopoli 1935, pp. 267–279; cf. Rödig, op. cit. (Fn.6), p. 172.

10 O. Weinberger, Bemerkungen zu J. Rödigs "Kritik des normlogischen Schließens", in: *Theory and Decision* 3 (1973), pp. 311–317.

11 J. Rödig, "Kritik des normlogischen Schließens", in: *Theory and Decision* 2 (1971), pp. 79–93.

mathematical logic to legal norms. According to Weinberger[12], for Tarski's conception of truth is based on the classical Aristotelian conception, namely a correspondence theory, but Tarski's formal semantics is developed independently of any particular interpretation of the notion of truth. In that system, logic only needs to obey the purely formal principle of bivalence, which asserts that each statement is either true or false. Weinberger refers to Tarski's special interpretation of the concept of truth, but the validity of his formal semantics is independent of this interpretation. Therefore, Tarski's formal semantics can be used to justify the logical treatment of legal inference in terms of the concept of truth-values.

I agree with Tammelo's and Rödig's view on the direct applicability of mathematical logic to law, which had also been advanced by Ulrich Klug as early as 1950.[13] Nowadays, the methods of mathematical logic are precisely developed and proven in such a way that one does not encounter essential methodological problems, as one does with deontic logic; one may rely on it and deal solely with its application, expecting great success.

3 The Quasi-Problem of Paradoxes in the Logic of Norms

Various attempts have been made to develop systems of a logic of norms, but unfortunately, none of the system developed is without its flaws. Objections are raised as to the paradoxes which arise from these respective systems. I hold the opinion that paradoxes in the logic of norms are always quasi-problems. Usually, they can be solved by consequent reasoning. I will propose to explain them as resulting from two sources: first, from a misinterpretation of logically correct formulae; secondly, from faulty formalizations of normative-logical reasoning.

12 O. Weinberger, op. cit. p. 312 f.
13 U. Klug, op. cit., especially p. 170.

By way of example, Alf Ross' paradox is given below.[14] Within the logic of norms, this paradox can be expressed as follows:

(1) $$Op \rightarrow O(p \vee q)$$

The reading of this formula is: If I shall post the letter, then I shall post it or burn it. The paradox expressed in formula (1) relates to the second class of paradoxes, which deals with faulty formalization. The problem concerns the formalization $O(p \vee q)$ which with regard to mathematical logic (especially standard predicate logic), is not well-formed. Following the formation rules of mathematical logic, no propositional operator is allowed to occur in an argument that is within the scope of a predicate. The question arises: why do we need this normative-logical formula and additional normative-logical formation rules for a deontic operator functioning as a sentential operator? An argument such as "he is lucky or I am lucky" can and should be formalized only as:

(2) $$La \vee Lb$$

But it must not be formalized as:

(3) $$L(a \vee b)$$

The consequence of formula (1) can and should be formalized only as:

(4) $$Op \vee Oq$$

14 A. Ross, "Imperatives and Logic," *Theoria* 7 (1941), p. 61 f. See also: D. Føllesdal and R. Hilpinen, "Deontic Logic: An Introduction," in: *Deontic Logic: Introductory and Systematic Readings* (R. Hilpinen, ed.), pp. 21–23.

The formula $O(p \vee q)$ gives the impression that both obligations, i.e., Op and Oq, are of equal importance.

One could introduce other formation rules than the ones of predicate logic'[15] but if one makes up rules which are not always in correspondence with the rules of predicate logic, then the system is more difficult to handle since it cannot rely on the tools of (standard) mathematical logic. Paradoxes may arise from faulty formalizations of the demonstrated kind, as well as from these together with logical calculi. Starting from a faulty formalization, it is not possible to interpret deductions correctly. In the present case, the faulty formalization does not produce the paradox, but it is the actual reason for generating the quasi-paradox.[16]

It could be argued that formula (4), which is logically well-formed, is still paradoxical. The Ross's Paradox, as it is called, can be formulated as follows:

(5) $$Op \rightarrow Op \vee Oq$$

The paradox expressed by formula (5) belongs to the first class of paradoxes, since it involves a misinterpretation of logical formulae. The semantic absurdity arising from the Ross's Paradox is a consequence of the inference from the formula $Op \vee Oq$ to the isolated formulas Op and Oq; it is believed that from the correctness of the obligation to post the letter (Op) the obligation to burn the letter (Oq) follows logically. This derivation, which has the character of an interpretation in human consciousness, is logically invalid.[17] Mathematical logic can clearly show the misinterpretation since the following formula is invalid:

15 Even if formula (3) is formed according to new formation rules, I cannot see the difference between formulae (2) and (3). If there is no difference in semantics, my question is (again): why do we need these normative-logical formation rules?

16 In addition to the formula $O\ (p \vee q)$ one should also do without the formalization $O\ (p \wedge q)$, $O\ (p \rightarrow q)$, $O\ (p \rightarrow q)$, etc.

(6) $$Op \vee Oq \rightarrow Oq$$

This can be easily seen by means of the shortcut truth-table method.[18]

Op	$\vee$	Oq	$\rightarrow$	Oq
+	+	–	–	–
3	2	3	1	2

The assignment of truth-values results in no inconsistency. So the formula is invalid. Oq is deducible from $Op \vee Oq$ only if $\neg Op$ is stated. But $\neg Op$ cannot be stated while Op is stated. If Op and Oq were stated at the same time, semantic absurdity could arise. As long as Oq is not stated, there is no paradox. If one reasons correctly, paradoxes are always quasi-problems, as opposed to true antinomies.

4 The Logical Provability from Applying Mathematical Logic to the Process of the Application of Law

Concerning my assertion that mathematical logic is directly applicable to law, I would like to give an example of a logical analysis of the process of the application of law and hence demonstrate its usefulness. Leo Reisinger has argued that the procedure of law application cannot be regarded as exact logical deduction, and gave an analysis utilizing a

17 Cf.: J. Rödig, "Über die Notwendigkeit einer besonderen Logik der Normen", op. cit., p. 184 f.

18 Cf.: I. Tammelo u. H. Schreiner, *Grundzüge und Grundverfahren der Rechtslogik I*, Pullach b. München 1974, pp. 33 ff.; I. Tammelo u. G. Moens, *Logische Verfahren der juristischen Begründung*, Wien-New York 1976, pp. 33–47.

uncertainty relation.[19] Although I recognize the great significance of his theory to the process of justification, I am of the opinion that the application of law can be logically analyzed, and that jurisdiction can (and should) be proven from legal assumptions using logical deduction. I share Rödig's view that this process constitutes a logical proof.[20] In my opinion, validity or correctness of a jurisdiction can be confirmed only if it is logically deduced from assumptions which are already considered valid. Observe that validity (correctness) or invalidity (incorrectness) can be assigned to both the assumptions and the conclusion in the same sense, provided one accepts an extended interpretation of the concepts of logical truth, i. e., "true" and "false."[21] As can easily be seen, jurisdiction is not directly deducible from the statute, even independently from the problem of stating the facts, for a norm of statute is too general and lacking in substance, unlike jurisdiction, which is concrete and rich in substance. A logical deduction does not allow deducing more than is already contained in the assumptions. In fact, someone who applies the statute interprets the law and forms a more or less subjective judgement, based on theories of legal science and other statements of jurisdiction. This judgement also contains value-judgements which usually remain implicit. However, value-judgements of this kind, i.e., applied theories of legal science and statements of jurisdiction should be explicitly marked. If these additional premises are explicitly described and employed, jurisdiction can be construed as a logical deduction devided from law and fact.

19 Leo Reisinger, "Probleme der logischen Struktur von Rechtsnormen und die Möglichkeiten des logischen Ausdrucks von unscharfen Rechtsbegriffen".

20 J. Rödig, *Die Theorie des gerichtlichen Erkenntnisverfahrens*, Berlin-Heidelberg-New York 1973, p. 3.

21 Cf. my work in Japanese: "Justice and Logic — The Role of Deductive Methods in Reasoning about Justice", in: *Justice — The Annual of Legal Philosophy* 1974, p. 51 f.

Subsequently, I would like to present a model of the logical structure of the application of law (see Table 1). In the present case, there is only one legal effect which is concrete, such that it needs no further concretization. Moreover, the judge only applies one construction of his own (which can be subsumed by a given theory of legal science).

Table 1: The logical structure of the process of law application: the predicate R stands for the legal requirements; predicate E stands for the legal effect.

(1) statute: $\forall x(R(x) \rightarrow E(x))$

(2) theories of legal science or statements of jurisdiction:

$$\forall x((R_1(x) \rightarrow R(x)) \wedge (R_2(x) \rightarrow R(x)) \wedge \ldots \wedge (R_n(x) \rightarrow R(x)))$$

(3) additional construction of the judge relating to the particular case:

$$\forall x(R_{1\cdot1}(x) \rightarrow R_1(x))$$

(4) fact: $R_{1\cdot1}(a)$

(5) jurisdiction : $E(a)$

The whole process of justification of law application can be represented in a one line logical formula as below:

$$\forall x((R(x) \rightarrow E(x)) \wedge \forall x((R_1(x) \rightarrow R(x)) \wedge (R_2(x) \rightarrow R(x)) \wedge \ldots \wedge (Rn(x) \rightarrow R(x))) \wedge \wedge \forall x(R_{1\cdot1}(x) \rightarrow R_1(x)) \wedge R_{1\cdot1}(a) \rightarrow E(a)$$

Rödig has also analyzed the process of law application as a result of logical deduction, trying thus to show its logical structure.[22] The formula presented above differs from his scheme[23] in at least one respect: in logically formalizing the process of putting a law in concrete form, Rödig treats the concretization as a definition of the requirement of the statute with legal consequence of the statute, precisely, as a definiens

22 J. Rödig, op. cit., pp. 163-184.

23 op. cit., p. 177 f.

which is connected to the concept by logical operator equivalence; whereas I represented the concretization by means of an additional assumption both in theories of legal science and statements of jurisdiction where the legal requirement of the statute is connected with concretized concepts by logical operator implication.[24] In my approach, it is not necessary to conceive of law and the application of law as a rigid system such as *Begriffsjurisprudenz* (conceptual jurisprudence); at the same time it is still possible to explain law application as logical proof.

The logical deduction of jurisdiction of the assumption in question can be shown as follows:

1.	$\forall x(R(x) \rightarrow E(x))$	
2.	$\forall x((R_1(x) \rightarrow R(x)) \wedge (R_2(x) \rightarrow R(x)) \wedge \ldots \wedge (R_n(x) \rightarrow R(x)))$	
3.	$\forall x(R_{1\cdot 1}(x) \rightarrow R_1(x))$	
4.	$R_{1\cdot 1}(a)$	$\underline{/\ E(a)}$
5.	$R(a) \rightarrow E(a)$	1, U.I.
6.	$(R_1(a) \rightarrow R(a)) \wedge (R_2(a) \rightarrow R(a))$ $\wedge \ldots \wedge (R_n(a) \rightarrow R(a))$	2, U.I.
7.	$R_{1\cdot 1}(a) \rightarrow R_1(a)$	3, U.I.
8.	$R_1(a) \rightarrow R(a)$	6, simpl.
9.	$R_1(a)$	7, 4, M.P.
10.	$R(a)$	8, 9, M.P.
11.	$E(a)$	5, 10, M.P.

Thus, we have shown that jurisdiction is deducible from legal assumptions; moreover, that the process of law application can be regarded as a logical proof and that mathematical logic is a useful tool for the analysis of problems related to legal theory. In short, the above analysis illustrates the fact that it is not necessary to develop and apply

24 I will deal with differences between these logical formalisms, as well as advantages and disadvantages of both formalisms elsewhere.

a special logic of norms.

I was of the opinion that additional inference rules are necessary in light of the existence of normative modalities, in the legal system, such as permission, prohibition, obligation, and so on.[25] Subsequently, I have observed that these notions are not definite in actual law; consequently, I am now convinced that the relations among these concepts cannot be stated formally as inference rules. Even in this respect, the logic of norms cannot be effectively applied in practice to the field of law.

The topics mentioned in this paper are intended as starting points in the quest for a legal logic. In the near future, some of them will be discussed in detail.

25 Cf. Y. Takeuchi and H. Yoshino, "Systeme und formelle Theorie der Gesetzgebung in Japan", in: *Studien zur Theorie der Gesetzgebung* (J. Rödig, ed.), Berlin - Heidelberg - New York 1976, p. 132.