The Logical Structure of Argumentation in Juridical Decisions

Hajime YOSHINO
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1 Introduction

Juridical argumentation has its own, unique logical structure. Legal logic, as the application of modern logic to the field of law and an essential part of philosophy of law, can and should analyze juridical argumentation by means of modern logic in order to reveal that logical structure.

When analyzing the logical structure of juridical argumentation, two dimensions of juridical argumentation have to be considered, i.e. the logical structure of the argument which justifies a given decision, and the

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logical structure of the line of reasoning which resulted in that decision. Consequently, we will deal with the logical structure of juridical justification on one hand, and with juridical decisions on the other.

Concerning juridical justification, one may assume, following Alexy\(^1\) (succeeding Wroblewski\(^2\)), two aspects of juridical justification: internal and external justification. Internal justification concerns the problem of whether a juridical decision can be logically deduced from the premises that are stated in order to justify that decision. In the case of external justification, the point is to show how the premises stated for internal justification are proven to be correct.\(^4\)

I maintain that a juridical decision is justified if it is deduced logically from premises already established, and hence considered correct, for the correctness of these established assumptions is transferable to the logically deduced conclusion. This justification is especially true for a judgement. Although a judgement usually is not directly (logically) deducible from a statute and the circumstances alone, when other assumptions, like constructions of a statute, are added the judgement is provable as a logical conclusion from the totality of premises.\(^4\) In this sense, internal justification can be conceived of as a logical deduction. The basic logical structure of such juridical argumentation is "modus ponens" of classical logic.\(^5\) Modus ponens is to be regarded as the basic reasoning scheme in juridical justification.

There are various possibilities for substantiation concerning external justification.\(^6\) But I am of the opinion that this kind of justification is a logical deduction, as well, or rather, a justification by means of logical deduction, as far as justification is concerned. The construction of a statute which is stated as a premise for internal justification is exactly justifiable only if it is logically deducible from other, already established, premises. The basis for constructing a statute could be statements on social habits, the economic situation, tradition, public opinion, etc. or other substantiating premises. A rule or criterion is needed in order to state these premises.\(^7\) The validity of the rule or criterion has to be presupposed for the justification of a premise, the statement of a construction of a statute. If these additional premises and rules, which are considered valid, are clearly stated, then this kind of justification can be proved by logical deduction; here, the rules should be thought of as premises.\(^8\) Therefore, from the logical point of view, it can be assumed that there is no essential difference between internal and external justification, since both deal with logical consequences.

\(^3\) Cf. R. Alexy, op. cit.; J. Wroblewski, op. cit.
\(^4\) This idea, together with a proof, I set forth at an international conference on logic of law in autumn 1976 in Salzburg, which was chaired by Professor Tammelo. My logical analysis of the process of of justification is published in the proceedings, see: H. Yoshino, Zu Anätzen der juristischen Logik, in I. Tammelo and H. Schreiner, (Hg.), Strukturierungen und Entscheidungen im Rechtsdenken. Wien-New York 1978, p. 283 f. (Yoshino (D)). A similar analysis is given in: Alexy, op. cit., pp. 278 f.

\(^5\) Cf. Yoshino (D)

\(^6\) See e.g. Alexy, op. cit., pp. 283 ff.

\(^7\) For an initial understanding in law, see the interesting analysis of Aarnio; A. Aarnio, Denkweisen der Rechtswissenschaft. Wien-New York 1979, pp. 123 ff. Also some contributions of the Symposium 'Argumentation in Legal Science' aimed at specifying such a rule (in formal terms), e.g.: in R. Alexy, Die Idee einer prozeduralen Theorie der juristischen Argumentation (in: Aarnio et al. (eds.), Methodologie und Erkennnistheorie der juristischen Argumentation, Rechtstheorie Beiheft 2, Berlin, 1981, pp. 177 ff);

\(^8\) I am of the opinion that the validity or correctness of the rule suggested by Alexy (see the contribution mentioned above) is required as a rule for purposes of external justification. If this rule is stated as valid and put forward clearly as a premise in the process of justification then—in my opinion—this process of external justification represents a logical consequence relation comparable to the process of internal justification.
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How can premises and rules be justified that are put forward for (external) justification of a construction of a statute which is itself a premise responsible for the justification of a decision? In order to answer this question, one could attempt to reduce the above mentioned process of justification step by step to more elementary premises. However, the validity of the most elementary premises cannot be established by this form of logical deduction, since they cannot be deduced from other already established premises. These premises have to be "determined" (decided) as valid (or correct). Consequently, a fundamental problem is how this decision (determination) is made in juridical argumentation. In other words, what is the logical structure of argumentation in juridical decisions? This problem has not yet been intensively analyzed from the logical point of view. Hence, in this work I want to concentrate my discussion on the problem of the logical structure of argumentation by which a juridical decision itself is reached.

2 The Method of Logical Analysis in Juridical Decisions

In order to logically analyze the logical structure of juridical argumentation in juridical decisions, one has to formulate an appropriate logical method.

There has been a fundamental disagreement concerning the methodology of legal logic. On one hand, it is argued that classical (mathematical) logic is not adequately applicable to the field of legal norms, and so efforts must be directed at developing a special logic of norms; thus many have attempted to develop such a special logic and have tried to apply it to legal science. On the other hand, others have argued that classical (mathematical) logic is adequately applicable to legal norms. Basically, I hold the latter opinion and consider the application of the logic of norms or deontic logic, as a particular type of logic of norms in the field of legal science, and especially the logical calculus of law, not suitable and sometimes problematic.

9 On this discussion of the method of legal logic, see my summary in: H. Yoshino, "Über die Notwendigkeit einer besonderen Normenlogik als Methode der juristischen Logik, in: U. Klug, u.a. (Hg.) Gesetzgebungstheorie, Juristische Logik, Zivil- und Prozeßrecht (Gedächtnisschrift für Jürgen Rößig) (Yoshino UD), p. 140, especially remark (2), (3) and (4).

10 Thorough and precise motivation for this opinion is given in my work referred to above. Cf. Yoshino (UD), pp. 140-161.

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This conception of truth is purely formal; consequently, the above principle can be reformulated as follows:

(A) \text{value}(\phi(a_1, ..., a_n), i) = 1 \text{ if and only if } \langle i(a_1), ..., i(a_n) \rangle \in i(\phi)

(B) \text{value}(\phi(a_1, ..., a_n), i) = 0 \text{ if and only if } \langle i(a_1), ..., i(a_n) \rangle \notin i(\phi)

In this way, truth-values 1 or 0 are assigned. If the assignment of an individual constant is an element of the set of individuals assigned to a predicate letter (e.g. \(i(a) \in i(\phi)\)), then the sentence (e.g. \(\phi(a)\)) is true, otherwise false. The truth-values 1 and 0 can be read indicatively true (\(A^*\)) or indicatively false (\(B^*\)), or normatively true (correct or valid) (\(A^{*\prime}\)) or normatively false (incorrect or invalid) (\(B^{*\prime}\)), dependent on the application domain of logic, as follows:

(A) \text{value}(P(t_1, ..., t_n), i) \text{ is indicatively true if and only if } \langle i(t_1), ..., i(t_n) \rangle \in i(P)

(B) \text{value}(P(t_1, ..., t_n), i) \text{ is indicatively false if and only if } \langle i(t_1), ..., i(t_n) \rangle \notin i(P)

(A) \text{value}(N(t_1, ..., t_n), i) \text{ is normatively true if and only if } \langle i(t_1), ..., i(t_n) \rangle \in i(N)

(B) \text{value}(N(t_1, ..., t_n), i) \text{ is normatively false if and only if } \langle i(t_1), ..., i(t_n) \rangle \notin i(N)

The logical calculus itself is not so important for development of a calculus. The logical calculus has only 1 and 0 as truth-values, so each predicate letter may have an assignment according to its own criterion. Hence, there is no problem of truth-value assignment with the immediate application of classical (mathematical) logic to norms because in the field of norms the principle of bivalence is valid, which uniquely assigns one of two possible values (i.e., 1 or 0) to each sentence. This principle is even valid in the case of so-called mixed premises, where, for instance, the antecedent of an implication formula is an indicative sentence, and the consequent is a sentence expressing a norm.

The logical formalization of a legal norm is expressible by means of the following formula of predicate logic, which expresses the legal norm: "The murderer shall be sentenced to death":

\[ \forall p (Mu(p) \rightarrow Sd(p)) \]

The reading of this formula is: for all \(p\), if \(p\) is a murderer, then \(p\) shall be sentenced to death.\(^{12}\) In this formalization the normative element of the consequence, i.e., the legal effect of the legal norm, is expressed by means of a predicate. Consequently, no problem of truth-value assignment ('1' or '0') results from this formalization of a legal norm.\(^ {13} \)

Thus, there is no problem of an immediate application of classical (mathematical) logic to legal norms as regards the assignment of truth-values. This kind of application is absolutely possible but more than that suitable and adequate, especially as it relates to the possibility of a formalization within a logical calculus.\(^ {14} \)


\(^{13}\) My formalization of legal norms within predicate logic has been criticized by Ota Weinberger. Cf. O. Weinberger, Kann man das normenlogische Folgerungssystem philosophisch begründen? (Überlegungen zu den Grundlagen des juristischen Folgerns), in: ARSP (Archiv für Rechts- und Sozialphilosophie), Vol. LXV/2 (1979) pp. 177 ff. I will try to give a precise response to his criticism elsewhere. At the world conference of IVR in Basel 1979, I refuted his criticism to some extent.
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Immediate application of classical (mathematical) logic has the advantage of avoiding the introduction of special formation and transformation rules peculiar to norms that are needed in various systems of deontic logic or logic of norms, and, hence, using the safe method of logic outlined above is recommended.

However, I don't want to dispense with the possibility of introducing these special rules, nor do I want to do without the possibility of extending the system of classical (mathematical) logic. I simply want to stress the possibility of impairing the reliability and practicability of the calculus by introducing such rules; take, for instance, the paradoxes of the logic of norms. If one abstracts from the problem of developing a calculus, one can certainly find advantages to such an extension and such a special way of formalization. The introduction of special rules may permit formulae to be stated in simpler terms which allows an easier reading. This kind of formalization can be helpful as a first step in an (exact) logical analysis, even if it is not precisely justifiable.

However, for an exact logical treatment these formulae should eventually be transformed to a precise formalization; in my opinion, a classical (mathematical) formalism is most adequate.

3 Popper's Falsificationism and Its Relevance to Scientific Reasoning

In order to clarify the logical structure of an argument leading to a juridical decision, one could compare this form of reasoning to explanations in natural sciences. Here Sir Karl Popper offers a logical analysis of scientific research; to be more concrete, he advances the so-called 'falsifiability thesis.' In order to apply this thesis to juridical argumentation in the next section, I want to describe Popper's thesis briefly.

In his book "The Logic of Scientific Discovery," Popper has shown the following: although reasoning in empirical science has so far been conceived of as induction, a general statement can never be proven by induction. The method which was thought of as induction can better be defined as "the deductive method of testing." "Theories are never em-

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15 The problem resulting from introducing special rules of formation and transformation to the logic of norms can be seen mainly in the paradoxes of the logic of norms. For a critical analysis of the paradoxes from this point of view, see: Yoshino (ID), pp. 155-158, and Yoshino (ID), pp. 280 ff.
16 This is the reason why I will use - for this purpose only - special operators different from the ones used in classical systems (cf. Subsection 4.3.).
17 I will try to do that in Subsection 4.3 of this paper.
19 op. cit., p. 30.
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One should deal with "falsifiability of a system rather than verifiability." In a way, Popper's method is a hypothetical-deductive method for testing by falsification. In this view, there are deductive relations between the statements of the theory. Universal empirical statements have the character of hypotheses, i.e., they are falsifiable by the falsification of a less universal statement. The inference rule in question, namely "the way in which the falsification of a conclusion entails the falsification of the system from which it is derived—is the modus tollens of classical logic." The logical structure of this inference can be formalized in propositional logic as follows:

\[(P \rightarrow Q) \land \neg Q \rightarrow \neg P\]

Which means: if \(Q\) is derivable from the system of sentences \(P\) (such that: \(P \rightarrow Q\) and if \(Q\) is falsified, then \(P\) is also falsified.

The hypothetical-deductive method for testing by falsification tests universal principles and theories, attempting to falsify them by examining less general, singular statements which may also confirm the principles and theories by experimentation and reflection. According to Popper, one should direct one's attention to the fact that a positive result in the test phase can only temporarily support the theory, for the theory can be overthrown by subsequent negative results. As long as the theory passes the deductive tests, it is said to be "corroborated."

20 op. cit., p. 40.
21 op. cit.
23 Cf. Popper, op. cit., p. 75.
24 op. cit., p. 76.
25 Cf. op. cit.

4 The Logical Structure of Argumentation in Juridical Decision Making

4.1 Proposal to Apply Popper's Falsifiability Thesis to Juridical Argumentation

I wish to propose applying Popper's thesis of hypothetical-deductive method for testing by falsification to juridical argumentation. In my opinion, the inference scheme "modus ponens," as the basic inference rule of reasoning is not only valid for the natural sciences, but also for social sciences, and hence for juridical argumentation.

In a former work, I discussed this with respect to argumentation in justice. Here, I want to argue that the inference rule of "modus tollens" and the hypothetical-deductive structure for testing by falsification are fundamental schemes of the logical structure of argumentation in juridical argumentation in general has been investigated extensively, e.g. K. Adomeit, Rechtsquellenfragen im Arbeitsrecht, München 1969; F. J. Säcker, Grundprobleme der kollektiven Koalitionsfreiheit, Düsseldorf 1969; P. Schmitt, Rechtswissenschaft und kritischer Rationalismus (I), in: RECHTSTHEORIE 2 (1971), pp. 67-94; (II) at the same place, pp. 24-44; also A. Podleck, Wertung und Werte im Recht, in: AgR 95 (1970), pp. 185-223. But, to the best of my knowledge, the application of Popper's falsifiability thesis to the field of juridical argumentation had not been studied thoroughly at the time of the publication of the German original version of my present work. Popper's theory has been thoroughly applied in the present paper.

26 op. cit., p. 33.
27 The application of the so-called critical rationalism to the field of law in general has been investigated extensively, e.g. K. Adomeit, Rechtsquellenfragen im Arbeitsrecht, München 1969; F. J. Säcker, Grundprobleme der kollektiven Koalitionsfreiheit, Düsseldorf 1969; P. Schmitt, Rechtswissenschaft und kritischer Rationalismus (I), in: RECHTSTHEORIE 2 (1971), pp. 67-94; (II) at the same place, pp. 24-44; also A. Podleck, Wertung und Werte im Recht, in: AgR 95 (1970), pp. 185-223. But, to the best of my knowledge, the application of Popper's falsifiability thesis to the field of juridical argumentation had not been studied thoroughly at the time of the publication of the German original version of my present work. Popper's theory has been thoroughly applied in the present paper.

28 H. Yoshino, Die Rolle der Logik in der Theorie des Rechts (contribution to the above mentioned world conference of IVR in 1979 (Yoshino IV)). The basic idea and analysis of applying the falsifiability thesis to law and to the theory of justice has already been demonstrated in my talk of November 1974 at the annual meeting of the Japanese Association of Legal Philosophy: H. Yoshino, Justice and Logic. They relevant paper was published as: The Role of Deductive Methods in Reasoning about Justice, in: Justice. The Annual of Legal Philosophy (1974), pp. 38-68.
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disputational decision.

It is sometimes said that a juridical decision is not obtained by
logical deduction, but rather by induction on each juridical experience
and based on a socio-economical basis. This assertion is partly true and
partly false. In my opinion, the inference rule of "modus tollens" as a
hypothetical-deductive method for testing by falsification is basic even
in the framework of "inductive reasoning" starting from particular
legal facts.

4.2 The Logical Structure of Argumentation in Juridical Decision
as "Modus Tollens"

Starting from a particular juridical experience, lawyers formulate a
general legal-normative statement, \( (N_l) \), or a legal-dogmatic theory,
\( (N_l) \), which contains the statement as a provisional hypothesis. Hence,
juridical experience is compared to certain statements, such as the code
of law, statements of jurisdiction and various assumptions which corre-
spond to the general sense of justice. Lawyers test the soundness of this
juridical-normative statement against the particular juridical-normative
statement \( (N_{l1}, N_{l2}, N_{l3}, ..., N_{lN}) \), which is deducible from the general
one. If a particular juridical-normative statement is negated \( (\neg N_{lN}) \),
then the general juridical-normative statement in question is also ne-
gated \( (\neg N_l) \). The logical structure of this form of argument is as
follows:

\[
(N_l \rightarrow N_{lN}) \land \neg N_{lN} \rightarrow \neg N_l
\]

This formula has the following reading: "If \( N_l \) is true then \( N_{lN} \) is true,
but if \( N_{lN} \) is false then the falsity of \( N_l \) is derivable from the conjunc-
tion of premises."

This formula is valid since it is an instance of "modus tollens."
The validity of this inference can also be confirmed by applying the
shortcut truth-table method; the assignment of truth-values yields a

\[
\begin{array}{cccccccc}
N_l & N_{lN} & N_{l1} & N_{l2} & N_{l3} & N_{l4} & N_{l5} & N_{l6} \\
\rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\
4 & 3 & 5 & 2 & 3 & 4 & 1 & 2 & 3 \\
\rightarrow & \rightarrow \uparrow & \uparrow & \uparrow
\end{array}
\]

Note, however, the following two points: First, in this formalization
the truth-values "1" and "0" are read normatively as "correct" or "in-
correct." Therefore, the negation (the negative normative assignment) of
the more general and the particular juridical-normative statements are
assigned the truth-value "0" (false), while the positive assignments have
the truth-value "1" (true). Second, all propositional letters in this for-

mula express normative statements in such a way that the assignment
of normatively read truth-values can be done uniformly. In light of the

\begin{align*}
\text{(A)} & : (A)'' \text{ and } (B)'' \text{ offer a semantic foundation for this formalism.}^{29}
\end{align*}

If the particular juridical-normative statement is not negated \( (N_{lN}) \),
then the logical structure of the argument is as follows:

\[
(N_l \rightarrow N_{lN}) \land N_{lN} \rightarrow N_l
\]

This inference is not logically valid; the application of the shortcut
truth-table method yields no contradiction in the assignment of truth-
values.

\[\text{29 For a description of the shortcut truth-table method, see e.g. I. Tam-
melto, H. Schreiner, Grundzüge und Grundverfahren der Rechtslogik,
vol. I, Pullach near Munich 1974, pp. 30 ff.}\]

\[\text{30 Cf. Section 2 of this contribution.}\]
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\[(N_1 \rightarrow N_{1,n}) \land N_{1,n} \rightarrow N_1\]
\[- + + + + - +
4 3 4 2 3 1 2\]

Thus, the more general juridical-normative statement is not proven, but temporarily corroborated. One cannot preclude the possibility that another particular juridical-normative statement \((N_{1,n+1})\), which is also deducible from the general statement, is evaluated negatively (falsified) and so the general statement is also evaluated negatively (falsified).

\[\neg (N_1 \rightarrow N_{1,n+1}) \land \neg N_{1,n+1} \rightarrow \neg N_1\]

Consequently, I am convinced that in juridical argumentation, decisions cannot and should not be seen as being based on verification, but rather on falsification of the general juridical-normative statement.\(^{31,32}\) In this way, the temporarily settled general juridical-normative statements are examined by several particular juridical-normative statements which are deducible from the former ones. This is done when an (important) falsification is encountered. A (general) juridical-normative statement is accepted, if it is sufficiently tested and not falsified; hence it is asserted that the juridical-normative statement is corroborated, thus, relatively correct. Eventually, it becomes accepted as a juridical decision.

\[^{31}\text{This is a starting point for inferring the problematic nature of the term "external justification." (cf. footnote 46 of this paper).}\]

\[^{32}\text{How is the falsification of a particular juridical-normative statement effected? I believe that the inference rule of "modus tollens" is essential here too, as long as falsification can be found. The most elementary (juridical-normative) statement, one that cannot be deduced by some reasoning process, has to be accepted or rejected as a subjective value judgement. These relations also hold for the effect evaluation as "modus tollens in the broad sense," to be explained in the next section.}\]

\[^{33}\text{This rule of inference ("modus tollens") and the scheme outlined of reasoning above are valid for modifications of juridical decision over time (cf., footnote 42 in this paper).}\]
4.3 The Logical Structure of Argumentation in Juridical Decisions as "Modus Tollens in the Broad Sense"

In Section 4.2, I described juridical decision-making as falsification-based by means of the inference rule "modus tollens." In this kind of falsification, the particular juridical-normative statement is deducible from the more general juridical-normative statement, whereby the latter is falsified if the former is falsified. Falsification of a juridical statement has another aspect of reasoning, which is not "modus tollens" in the exact classical sense of logic, but very similar. I will call this form of reasoning "modus tollens in the broad sense." This kind of reasoning applies mainly to falsification as performed by effect evaluation.  34

The logical structure of argumentation in juridical decision-making by means of effect evaluation is as follows: If a juridical-normative statement or a juridical-normative theory (Nn1) is accepted, the effects (E1, E2, E3, ..., En, n) follow as a consequence of the application of such a juridical-normative statement. Some of these results are evaluated negatively. Hence, one can conclude that the original juridical-normative statement has to be evaluated negatively. This line of reasoning has a structure similar to falsification by means of "modus tollens." The logical structure of this inference can be represented as follows:

\[(Nn1 \rightarrow E_{11}) \land (Nn1 \rightarrow E_{12}) \land ... \land (Nn1 \rightarrow E_{1n}) \land \lnot E_{1n} \rightarrow \lnot Nn1\]

Falsification here concerns the subformula:

\[(Nn1 \rightarrow E_{1n}) \land \lnot E_{1n} \rightarrow \lnot Nn1\]

The reading of this formula is: "From the acceptance of the juridical-normative statement (Nn1) the effect (E1n, n) follows as a result of the application of the statement. Since the effect is evaluated normatively-negative (\lnot E1n, n), it follows that the acceptance of the juridical-normative statement is evaluated normatively-negative (\lnot Nn1)." This inference is close in its effect to "modus tollens." Below, I will call such an inference rule "modus tollens in the broad sense."

The entire process of argumentation of juridical decision as performed by "modus tollens in the broad sense," which leads to a (temporarily) corroborated juridical-normative statement, could be described in the same way as above (6) – (8) by means of "modus tollens in the proper sense, but this is not done here.  35

In the formalizations (9) and (10) above, the special operators "\lnot\" and "\rightarrow\" are used to make this kind of reasoning more transparent.  36 Our formalization deviates from the formalization of classical (mathematical) logic. In order to prove that the inference "modus tollens in the broad sense" is valid and in order to develop a logical calculus for the application of classical (mathematical) logic to juridical argumentation, we need to transform the (extended) schemes (9) and (10) to classical (mathematical) logic.  37 This will be done below.

This transformation requires consideration of two problems. On one hand, there is the problem of formalizing the falsification, i.e., the normatively negative evaluation of results. The propositional letter "E1n, n" in the first conjunct of the antecedent should be evaluated as

34 For a discussion of effect evaluation in legal science, cf., A. Podlech, pp. 185-223, especially p. 201.

35 A sketch of this form of argument would correspond to the schemes (6) – (8) in this paper.

36 These operators are introduced for only one reason: the usage of these operators shows the logical structure of the argumentation we are concerned with; they are simpler and therefore easier to understand. In order to develop a logical calculus, we would need to extend the formalization rules and give a fixed definition of these operators, in the sense of a semantical justification. I do not want to deal with this problem here, as it is not essential in enhancing the readability of the formulae.

37 See Section 2 in this paper.
indicatively true, while the same letter in the second conjunct (of the antecedent) should be evaluated as normatively false. Within logic, evaluating the same logical symbol as indicatively true or false in one instance, and normatively true or false in another is not permissible. Being aware of this problem, I used different symbols in the formulas (9) – (10) and (3) – (5). In the latter formulae, each propositional letter can be uniformly evaluated by normative truth such that no problems occur with negation.

On the other hand, there is the problem of how to formalize the effect of applying a norm. Hence, I used different symbols for implication in the formulae (9) – (10) and (3) – (5). In order to correctly formalize the effect of applying a norm, we would have to solve the problem of formalizing causality.

When giving a precise formalization of "modus tollens in the broad sense," I do not preclude the possibility of formalizing the logical structure of juridical falsification by effect evaluation utilizing the inference rule "modus tollens" in a precise way. This could be done by introducing additional formation and transformation rules. However, I can see the possibility of reformulating the formalization to classical (mathematical) logic by adding a universal assumption, which is peculiar to juridical practice and captures the meaning of "modus tollens in the broad sense." By way of example, below I attempt such a reformulation along the lines of the aforementioned argument shown just.

I am convinced that the following assumption is tacitly accepted in juridical argumentation:

(11)  "If the application of a juridical-normative statement (legal norm and juridical theory included) results in an effect which is to be evaluated negatively, then the juridical-normative statement is also to be evaluated negatively."

This assumption should not be considered to be a logical rule, but a premise in logical reasoning. If this assumption is added as a supplementary assumption, then juridical falsification by effect evaluation, i.e., "modus tollens in the broad sense," can be reconstructed as a classical, logically valid inference as described below:

The following letters are used in the formulae:

\[
N(\cdot) \text{ is a juridical-normative statement (legal norm and juridical theory included)} \\
P(\cdot) \text{ is a phenomenon} \\
Rs(\cdot, ...) \text{ application of } \cdot \text{ results in } .. \\
Ne(\cdot) \text{ should be evaluated negatively}^{39}
\]

(12) \( \forall n \forall s \left( N(n) \land P(s) \land Rs(n,s) \rightarrow ((P(s) \rightarrow Ne(s)) \rightarrow Ne(n)) \right) \)
(13) \( N(n) \land P(s) \land Rs(n,s) \)
(14) \( P(s) \land Ne(s) \)
(15) \( Ne(n) \)

This inference can be rewritten as the following formula:

(16) \( (\forall n \forall s (N(n) \land P(s) \land Rs(n,s) \rightarrow ((P(s) \rightarrow Ne(s)) \rightarrow Ne(n)))) \land \)
\( (N(n) \land P(s) \land Rs(n,s)) \land (P(s) \land Ne(s)) \rightarrow Ne(n) \)

The (implicational) formula (16) — with (12) – (14) as antecedents and (15) as conclusion — is logically valid, which (12) encodes assumption (11).

Logical validity can be proven as follows:

1. \( \forall n \forall s (N(n) \land P(s) \land Rs(n,s) \rightarrow ((P(s) \rightarrow Ne(s)) \rightarrow Ne(n))) \)

Note that the "obligatory aspect" of statement (11) is covered by the predicate (cf., formalization (I)).
In this way, the logical structure of juridical falsification by "modus
tollens in the broad sense" can be reconstructed within classical
(mathematical) logic. But I do not assert that this reformulation is the best
solution; it is only one possible solution. I will seek a better solution in
a future publication.

Finally, it should be stressed again that the inference rules of "modus
tollens" mentioned above, i.e., "modus tollens" proper, and "modus
tollens in the broad sense" are basic schemes of juridical argumentation
in legal decision-making. Nowadays, it is the prevailing opinion in both
the Federal Republic of Germany and Japan that juridical decision-
making is not direct deduction from statutes, but is obtained between
statutes and individual circumstances by the "constant interaction, a
to-and-fro movement of one’s focus" (K.Entisch) or "bringing-to-accor-
dance" (Arthur Kaufmann). This relation of "constant interaction, the
to-and-fro movement of one’s focus" can be considered to be a logical

40 K. Entisch, Logische Studien zur Gesetzesanwendung, Heidelberg 1942,
second edition 1960, p. 15.
41 Arthur Kaufmann, Analogie und "Natur der Sache," Zöglich ein
Beitrag zur Lehre vom Typus, Karlsruhe 1965, p. 29; also published in:
A. Kaufmann, Rechtsphilosophie im Wandel. Stationen eines Weges,
5 Logical Analysis of Argumentation in Juridical Decision

In my opinion, several decisions involve argumentations that have the logical structure of "modus tollens" or "modus tollens in the broad sense." This has already been demonstrated in a logical analysis of Japanese jurisdiction. Subsequently, I want to give an example of jurisdiction from Germany, namely the Bundesgerichtshof (German Federal Court of Justice); thereby, I will analyze the logical structure in order to prove, for instance, that juridical decision has the logical structure of "modus tollens in the broad sense."

BGHSt 25, 30. This jurisdiction is concerned with the element of "using signs" within the meaning of § 86a (1) of the German Criminal Code.

The relevant law says:

§ 86a The German Criminal Code:

"Use of signs of anti-constitutional organizations:
(1) Whoever propagates in public signs of one of the parties or associations specified in § 86 (1), (2), and (4) within the spatial purview of this law, shall be punished with up to three years of imprisonment or penalty."

In the reasons of this judgement, wherein the facts are of no importance for this logical analysis, the following argumentation can be found (the labels are mine, H.Y.):

"§ 86a Criminal Code does not contain as an element of the of-

43 BGHSt = Entscheidungen des BGH in Strafsachen (Decisions of the Federal Court of Justice in Criminal Cases).
44 § 86a (1) has been reformulated by an act of parliament on October 28, 1994 (BGBl. I, 3196)."
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statement (19), where the court demonstrates the negative evaluation of the effects, following the Select Committee on Penal Reform. If one tries to formalize the above argumentation precisely—thereby considering the subjunctive and the partly implicitly given negative evaluations—then the following formulae result:

\[(17') \quad (Na_1 \Rightarrow E_1) \land -E_1\]
\[(18') \quad ((Na_1 \Rightarrow E_{1.1}) \land (Na_1 \Rightarrow E_{1.2})) \land (-E_{1.1} \land -E_{1.2})\]
\[(20') \quad (Na_1 \Rightarrow E_2) \land -E_2\]

It is obvious from the reasons given above that the construction of the statute leading to these effects is evaluated negatively even in the judgement, as a consequence of the argument. The logical formula corresponding to the negative evaluation of the construction of the statute under consideration is:

\[(21') \quad -Na_1\]

Hence the above argumentation can be formalized as a whole as follows:

\[(17' - 21') \quad ((Na_1 \Rightarrow E_1) \land -E_1) \land ((Na_1 \Rightarrow E_{1.1}) \land (Na_1 \Rightarrow E_{1.2}) \land (-E_{1.1} \land -E_{1.2}) \land (Na_1 \Rightarrow E_2) \land -E_2) \rightarrow -Na_1\]

This formula typically has the logical structure of "modus tollens in the broad sense." This formula can be decomposed into elementary formulae, each of which contains a "modus tollens in the broad sense." \(^{45}\)

\(^{45}\) As seen in the analysis of the reasons in the present judgement, the effect evaluation is not performed only once but several times. From the logical point of view a single falsification is sufficient, especially with proper "modus tollens." In the reasons of a judgement it is recommendable to present the falsification of more effects of the application (of a juridical-normative statement) for two reasons: on one hand, falsification (normatively negative evaluation) of the effect of a juridical-normative

6 Conclusion

In this paper, I have aimed to clarify the logical structure of argumentation in juridical decisions. The theses for which I argued and which are partly proven can be succinctly summarized as follows:

1. Justification is a question of logical consequence. This is not only valid for so-called internal justification, but for so-called external justification as well. In this respect, there is no essential difference between the two.\(^{46}\)

\(^{46}\) In my opinion, it would not be adequate then to use the term "justification" for argumentation in the sense of "external justification" because in this kind of argumentation decision is done, that is, the acceptance of the juridical-normative statement, and this decision is not finitely (logically) justifiable; moreover, it cannot be "verified" but only "falsified".

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2. The justification of the ultimately elementary stated assumption in juridical decision cannot be done by deduction from other statements, but must be determined (or decided).

3. In order to analyze the logical structure of juridical argumentation, the method of classical (mathematical) logic can be adequately applied.

4. To make the logical structure of juridical decisions clear, Popper's falsifiability thesis should be introduced to juridical argumentation.

5. I propose the scheme of falsification by "modus tollens" as a basic scheme of argumentation in juridical decision.

6. On one hand, the logical structure of a juridical decision can be captured in an exact logical sense, by "modus tollens" in the proper sense, where a particular juridical-normative statement is deduced from a more general juridical-normative statement (including a legal norm or juridical theory). On the other hand, the logical structure of a juridical decision can be captured in another sense, as "modus tollens in the broad sense," whereby effect evaluation of the application of a juridical-normative statement are taken to considered. In both cases, we presented a formalization of the respective argumentations.

7. The logical structure of a juridical decision as "modus tollens in the broad sense" has been proven by way of example of an analysis of a judgement of the German Federal Court of Justice.

8. As a result of the above, analyses and proofs, it can be finally argued that one should direct one's attention more to the problem of falsification and falsifiability in juridical argumentation (as opposed to external justification and external justifiability).