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1 Introduction

Juridical argumentation has its own, unique logical structure. Legal logic, as the application of modern logic to the field of law and an essential part of philosophy of law, can and should analyze juridical argumentation by means of modern logic in order to reveal that logical structure.

When analyzing the logical structure of juridical argumentation, two dimensions of juridical argumentation have to be considered, i.e. the logical structure of the argument which justifies a given decision, and the

* This work is an extension of a talk given at the symposium in Helsinki (10.-12. December 1979) on "Argumentation in Legal Science", which was supervised by Professor Dr. *Aarnio*. The original paper of this work was published in German as: "Die logische Struktur der Argumentation bei der juristischen Entscheidung", in Aarnio, A., (Hrsg.), Methodologie und Erkenntnistheorie der Argumentation (Rechtstheorie Beiheft 2), Berlin (Dunker & Humblot Verlag) 1981, S. 235 ff. In the summer of 1996, I had the opportunity to study at the Institute for Legal Philosophy and Legal Information Science at the University of Munich as a Humboldt-fellow. At that time, I worked on this revised English version. I thank my hosts, Professor Lothar Philipps, Mr. Helmut Prendinger, who helped me with the translation, and Mr. Frank Höfinger, my research assistant, for their kind help and support.

logical structure of the line of reasoning which resulted in that decision. Consequently, we will deal with the logical structure of juridical justification on one hand, and with juridical decisions on the other.

Concerning juridical justification, one may assume, following *Alexy*[1] (succeeding *Wróblewski*[2]), two aspects of juridical justification: internal and external justification. Internal justification concerns the problem of whether a juridical decision can be logically deduced from the premises that are stated in order to justify that decision. In the case of external justification, the point is to show how the premises stated for internal justification are proven to be correct.[3]

I maintain that a juridical decision is justified if it is deduced logically from premises already established, and hence considered correct, for the correctness of these established assumptions is transferable to the logically deduced conclusion. This justification is especially true for a judgement. Although a judgement usually is not directly (logically) deducible from a statute and the circumstances alone, when other assumptions, like constructions of a statute, are added the judgement is provable as a logical conclusion from the totality of premises.[4] In this sense, internal justification can be conceived of as a logical deduction. The basic logical structure of such juridical argumentation is "modus

1 Cf. *R. Alexy*, Theorie der juristischen Argumentation. Frankfurt am Main 1978, pp. 273 ff.

2 Cf. *J. Wrôblewski*, Legal Syllogism and Rationality of Judical Decision, in: RECHTS-THEORIE 5 (1974), pp. 39 ff.

3 Cf. *R. Alexy*, op. cit.; *J. Wrôblewski*,op. cit.

4 This idea, together with a proof, I set forth at an international conference on logic of law in autumn 1976 in Salzburg, which was chaired by Professor Tammelo. My logical analysis of the process of of justification is published in the proceedings, see: *H. Yoshino*, Zu Anätzen der juristischen Logik, in I. Tammelo and H. Schreiner, (Hg.), Strukturierungen und Entscheidungen im Rechtsdenken, Wien-New York 1978, p. 283 f. (*Yoshino (I)*), A similar analysis is given in: *Alexy*, op. cit., pp. 278 f.

ponens" of classical logic.[5] Modus ponens is to be regarded as the basic reasoning scheme in juridical justification.

There are various possibilities for substantiation concerning external justification.[6] But I am of the opinion that this kind of justification is a logical deduction, as well, or rather, a justification by means of logical deduction, as far as justification is concerned. The construction of a statute which is stated as a premise for internal justification is exactly justifiable only if it is logically deducible from other, already established, premises. The basis for constructing a statute could be statements on social habits, the economic situation, tradition, public opinion, etc. or other substantiating premises. A rule or criterion is needed in order to state these premises.[7] The validity of the rule or criterion has to be presupposed for the justification of a premise, the statement of a construction of a statute. If these additional premises and rules, which are considered valid, are clearly stated, then this kind of justification can be proved by logical deduction ; here, the rules should be thought of as premises.[8] Therefore, from the logical point of view, it can be assumed that there is no essential difference between internal and external justification, since both deal with logical consequences.

5 Cf. *Yoshino (I)*

6 See e. g. *Alexy*, op. cit., pp. 283 ff.

7 For an initial underatanding in law, see the interesting analysis of Aarnio; *A. Aarnio*, Denkweisen der Rechtswissenschaft, Wien-New York 1979, pp. 123 ff. Also some contributions of the Symposium 'Argumentation in Legal Science' aimed at specifying such a rule (in formal terms), e.g. : in *R. Alexy*, Die Idee einer prozeduralen Theorie der juristischen Argumentation (in: *Aarnio* et al. (eds.), Methodologie und Erkenntnistheorie der juristischen Argumentation, Rechtstheorie Beiheft 2, Berlin, 1981, pp. 177 ff).

8 I am of the opinion that the validity or correctness of the rule suggested by Alexy (see the contribution mentioned above) is required as a rule for purposes of external justification. If this rule is stated as valid and put forward clearly as a premise in the process of justification then—in my opinion—this process of external justification represents a logical consequence relation comparable to the process of internal justification.

How can premises and rules be justified that are put forward for (external) justification of a construction of a statute which is itself a premise responsible for the justification of a decision? In order to answer this question, one could attempt to reduce the above mentioned process of justification step by step to more elementary premises. However, the validity of the most elementary premises cannot be established by this form of logical deduction, since they cannot be deduced from other already established premises. These premises have to be "*determined*" (decided) as valid (or correct). Consequently, a fundamental problem is how this decision (determination) is made in juridical argumentation. In other words, what is the logical structure of argumentation in *juridical decisions*? This problem has not yet been intensively analyzed from the logical point of view. Hence, in this work I want to concentrate my discussion on the problem of the logical structure of argumentation by which a juridical decision itself is reached.

2 The Method of Logical Analysis in Juridical Decisions

In order to logically analyze the logical structure of juridical argumentation in juridical decisions, one has to formulate an appropriate logical method.

There has been a fundamental disagreement concerning the methodology of legal logic. On one hand, it is argued that classical (mathematical) logic is not adequately applicable to the field of legal norms, and so efforts must be directed at developing a special logic of norms; thus many have attempted to develop such a special logic and have tried to apply it to legal science. On the other hand, others have argued that classical (mathematical) logic is adequately applicable to legal norms.[9] Basically, I hold the latter opinion and consider the application of the logic of norms or deontic logic, as a particular type of logic of norms in the field of legal science, and especially the logical calculus of law, not suitable and sometimes problematic.[10]

Below, I shall argue briefly that classical (mathematical) logic is immediately applicable and develop the necessary formalization to implement it.

Since the main criticism against the immediate application of classical logic is that the truth-values of classical logic are not suitable for norms, it is necessary to show the direct applicability of classical (mathematical) logic to the field of norms in semantic terms. To do this, one may appeal to the following scheme deriving from the formal semantics of Tarski.[11]

(A) $\phi(\alpha_1, \ldots, \alpha_n)$ is true with respect to i if and only if $\langle i(\alpha_1), \ldots, i(\alpha_n)\rangle \in i(\phi)$

(B) $\phi(\alpha_1, \ldots, \alpha_n)$ *is false with respect to* i if and only if $\langle i(\alpha_1), \ldots, i(\alpha_n)\rangle \notin i(\phi)$

---

9 On this discussion of the method of legal logic, see my summary in: *H. Yoshino*, Über die Notwendigkeit einer besonderen Normenlogik als Methode der juristischen Logik, in: U. Klug, u.a. (Hg.) Gesetzgebungstheorie, Juristische Logik, Zivil-und Prozeßrecht (Gedächtnisschrift für Jürgen Rödig) (*Yoshino (II)*), p. 140, especially remark (2), (3) and (4).

10 Thorough and precise motivation for this opinion is given in my work referred to above. Cf. *Yoshino (II)*, pp. 140-161.

11 This kind of semantic foundation is also explained in *Yoshino (II)*, pp. 144-147. For Tarski's semantic foundation, see two of his papers: *A. Tarski*, Der Wahrheitsbegriff in den formalisierten Sprachen, in: Studia Philosophica Commentarii Philosophicae Polonorum I, Leopoli (Lemberg) 1935, pp. 261-405, reprint in: K. Berker and L. Kreiser, (Hg.): Logik-Texte, Berlin (Ost) 1971, pp. 447-559, especially pp. 480-488; *Tarski*, The Semantic Conception of Truth and the Foundations of Semantics, in: Journal of Philosophy and Phenomenological Research 4 (1944), pp. 341-375, reprint in: L. Linsky, (Hg.): Semantics and the Philosophy of Language, Urbana (Ill.), 1952, pp. 13-47. For the system of this semantics, see e.g.: *F. von Kutschera, A. Breitkopf*, Einführung in die moderne Logik, Freiburg–München 1971, pp. 86-90; a thorough account is given in: *W. Stegmüller*, Das Wahrheitsproblem und die Idee der Semantik, Wien 1957. The formalization of *Tarski's* definition of a metalinguistic conception of truth (interpretational semantics) in the pre-

This conception of truth is purely formal; consequently, the above principle can be reformulated as follows:

(A)′ $value(\phi(\alpha_1, \ldots, \alpha_n), i) = 1$ if and only if
$\langle i(\alpha_1), \ldots, i(\alpha_n)\rangle \in i(\phi)$

(B)′ $value(\phi(\alpha_1, \ldots, \alpha_n), i) = 0$ if and only if
$\langle i(\alpha_1), \ldots, i(\alpha_n)\rangle \notin i(\phi)$

In this way, truth-values 1 or 0 are assigned. If the assignment of an individual constant is an element of the set of individuals assigned to a predicate letter (e.g. $i(\alpha_1) \in i(\phi)$), then the sentence (e.g. $\phi(\alpha_1)$) is true, otherwise false. The truth-values 1 and 0 can be read indicatively true (A″) or indicatively false (B″), or normatively true (correct or valid) (A‴) or normatively false (incorrect or invalid) (B‴), dependent on the application domain of logic, as follows:

(A)″ $value(P(t_1, \ldots, t_n), i)$ is indicatively true if and only if
$\langle i(t_1), \ldots, i(t_n)\rangle \in i(P)$

(B)″ $value(P(t_1, \ldots, t_n), i)$ is indicatively false if and only if
$\langle i(t_1), \ldots, i(t_n)\rangle \notin i(P)$

(A)‴ $value(N(t_1, \ldots, t_n), i)$ is normatively true if and only if
$\langle i(t_1), \ldots, i(t_n)\rangle \in i(N)$

(B)‴ $value(N(t_1, \ldots, t_n), i)$ is normatively false if and only if
$\langle i(t_1), \ldots, i(t_n)\rangle \notin i(N)$

The reading itself is not so important for development of a calculus. The logical calculus has only 1 and 0 as truth-values, so each predicate

sent work is based essentially on the presentation of F. von Kutschera (*Kutschera*, op. cit., p. 89) and of P. Hinst (*P. Hinst*, Wahrheit und Bedeutung. Vorschläge zu einem fundamental semantischen Aufbau von Wissenschaftssprachen, München 1974 (unpublished), p. 19).

letter may have an assignment according to its own criterion. Hence, there is no problem of truth-value assignment with the immediate application of classical (mathematical) logic to norms because in the field of norms the principle of bivalence is valid, which uniquely assigns one of two possible values (i.e., 1 or 0) to each sentence. This principle is even valid in the case of so-called *mixed* premises, where, for instance, the antecedent of an implication formula is an indicative sentence, and the consequent is a sentence expressing a norm.

The logical formalization of a legal norm is expressible by means of the following formula of predicate logic, which expresses the legal norm: "The murderer shall be sentenced to death":

(1) $$\forall p(Mu(p) \rightarrow Sd(p))$$

The reading of this formula is: for all p, if p is a murderer, then p shall be sentenced to death.[12] In this formalization the normative element of the consequence, i.e., the legal effect of the legal norm, is expressed by means of a predicate. Consequently, no problem of truth-value assignment ('1' or '0') results from this formalization of a legal norm.[13]

Thus, there is no problem of an immediate application of classical (mathematical) logic to legal norms as regards the assignment of truth-values. This kind of application is absolutely possible but more than that suitable and adequate, especially as it relates to the possibility of a formalization within a logical calculus.[14]

12 For this kind of formalization, cf. *U. Klug*, Juristische Logik, third edition, p. 51, and *Yoshino (II)*, p. 145.

13 My formalization of legal norms within predicate logic has been criticized by Ota Weinberger. Cf. *O. Weinberger*, Kann man das normenlogische Folgerungssystem philosophisch begründen? (Überlegungen zu den Grundlagen des juristischen Folgerns), in: ARSP (Archiv für Rechts- und Sozialphilosophie), Vol. LXV/2 (1979) pp. 177 ff. I will try to give a precise response to his criticism elsewhere. At the world conference of IVR in Basel 1979, I refuted his criticism to some extent.

Immediate application of classical (mathematical) logic has the advantage of avoiding the introduction of special formation and transformation rules peculiar to norms that are needed in various systems of deontic logic or logic of norms, and, hence, using the safe method of logic outlined above is recommended.

However, I don't want to dispense with the possibility of introducing these special rules, nor do I want to do without the possibility of extending the system of classical (mathematical) logic. I simply want to stress the possibility of impairing the reliability and practicability of the calculus by introducing such rules; take, for instance, the paradoxes of the logic of norms.[15] If one abstracts from the problem of developing a calculus, one can certainly find advantages to such an extension and such a special way of formalization. The introduction of special rules

Cf. *H. Yoshino*, Logische Struktur der Rechtsnorm (a contribution to the above mentioned conference, *Yoshino (III)*). Concerning his criticism of my formalism of legal norms within predicate logic given in this paper, I want to stress only the following points.

1. It is semantically justifiable to interpret formula (1) as a whole as normatively true or false [this concerns his criticism on p. 179].

2. This formalization within a logical calculus is suitable, independently of the normative interpretation of the logical concept of truth; following the standards set by the literature, logical operations are only concerned about the truth-values 1 and 0. Therefore, the problem of formalization within a logical calculus neither depends on the decision nor on the answer to the question whether the formula expressing facts relevant for a legal norm (e.g. '*Mu*(.)') is an indicative or a normative sentence, or whether this formula should be assigned different truth-values as is the case for the legal effects (e. g. '*Sd*(.)') [this concerns his criticism on p. 178].

14 The suitability and adequacy of the application of classical (mathematical) logic to legal norms can be tested by comparising it to the logic of norms, by means of analyzing the legal norm and the juridical argument using both methods, and then comparing the results of both methods. This has been done by me in a paper dealing with so-called "contrary-to-duty imperatives." See: *Yoshino (II)*, p. 151-157.

may permit formulae to be stated in simpler terms which allows an easier reading. This kind of formalization can be helpful as a first step in an (exact) logical analysis, even if it is not precisely justifiable.[16] However, for an exact logical treatment these formulae should eventually be transformed to a precise formalization; in my opinion, a classical (mathematical) formalism is most adequate.[17]

3 Popper's Falisificationism and Its Relevance to Scientific Reasoning

In order to clarify the logical structure of an argument leading to a juridical decision, one could compare this form of reasoning to explanations in natural sciences. Here Sir Karl Popper offers a logical analysis of scientific research; to be more concrete, he advances the so-called 'falsifiability thesis.' In order to apply this thesis to juridical argumentation in the next section, I want to describe Popper's thesis briefly.

In his book "The Logic of Scientific Discovery,"[18] Popper has shown the following: although reasoning in empirical science has so far been conceived of as induction, a general statement can never be proven by induction. The method which was thought of as induction can better be defined as "the deductive method of testing."[19] "Theories are never em-

15 The problem resulting from introducing special rules of formation and transformation to the logic of norms can be seen mainly in the paradoxes of the logic of norms. For a critical analysis of the paradoxes from this point of view, see: *Yoshino (II)*, pp. 155-158, and *Yoshino(I)*, pp. 280 ff.

16 This is the reason why I will use – for this purpose only – special operators different from the ones used in classical systems (cf. Subsection 4.3.).

17 I will try to do that in Subsection 4. 3 of this paper.

18 *K. R. Popper*, The Logic of Scientific Discovery, London 1959, third edition 1962.

19 op. cit., p. 30.

pirically verifiable"[20] and one should deal with "*falsifiability* of a system rather than verifiability."[21] In a way, Popper's method is a *hypothetical-deductive method for testing by falsification.*[22] In this view, there are deductive relations between the statements of the theory. Universal empirical statements have the character of hypotheses, i.e., they are falsifiable by the falsification of a less universal statement.[23] The inference rule in question, namely "the way in which the falsification of a conclusion entails the falsification of the system from which it is derived—is the *modus tollens* of classical logic."[24] The logical structure of this inference can be formalized in propositional logic as follows:

$$(P \rightarrow Q) \wedge \neg Q \rightarrow \neg P \qquad (2)$$

Which means: if Q is derivable from the system of sentences P (such that: $P \rightarrow Q$) *and if* Q is falsified, then P is also falsified.[25]

The hypothetical-deductive method for testing by falsification tests universal principles and theories, attempting to falsify them by examining less general, singular statements which may also confirm the principles and theories by experimentation and reflection. According to Popper, one should direct one's attention to the fact that a positive result in the test phase can only temporarily support the theory, for the theory can be overthrown by subsequent negative results. As long as the theory passes the deductive tests, it is said to be "corroborated."[26]

20 op. cit., p. 40.
21 op. cit.
22 For this characterization, cf. *H. Sakamoto, H. Sakai*, Modern Logic (Japanese), new edition, Tokyo 1970, p. 22 f.
23 Cf. *Popper*, op. cit., p. 75.
24 op. cit., p. 76.
25 Cf. op. cit.

4 The Logical Structure of Argumentation in Juridical Decision Making

4.1 Proposal to Apply Popper's Falsifiability Thesis to Juridical Argumentation

I wish to propose applying Popper's thesis of hypothetical-deductive method for testing by falsification to juridical argumentation.[27] In my opinion, the inference scheme "modus ponens," as the basic inference rule of reasoning is not only valid for the natural sciences, but also for social sciences, and hence for juridical argumentation.

In a former work,[28] I discussed this with respect to argumentation in justice. Here, I want to argue that the inference rule of "modus tollens" and the hypothetical-deductive structure for testing by falsification are fundamental schemes of the logical structure of argumentation in juridi-

26 op. cit., p. 33.
27 The application of the so-called *critical rationalism* to the field of law in general has been investigated extensively, e.g. *K. Adomeit*, Rechtsquellenfragen im Arbeitsrecht, München 1969; *F. J. Säcker*, Grundprobleme der kollektiven Koalitionsfreiheit, Düsseldorf 1969; *P. Schwerdtner*, Rechtswissenschaft und kritischer Rationalismus (I), in: RECHTSTHEORIE 2 (1971), pp. 67-94; (II) at the same place, pp. 24-44; also *A. Podlech*, Wertung und Werte im Recht, in: AöR 95 (1970), pp. 185-223. But, to the best of my knowledge, the application of Popper's falsifiability thesis to the field of juridical argumentation had not been studied thoroughly at the time of the publication of the German original version of my present work. Popper's theory has been throughly applied in the present paper.
28 *H. Yoshino*, Die Rolle der Logik in der Theorie des Rechts (contribution to the above mentioned world conference of IVR in 1979 (*Yoshino (IV)*). The basic idea and analysis of applying the falsifiability thesis to law and to the theory of justice has already been demonstrated in my talk of November 1974 at the annual meeting of the Japanese Association of Legal Philosophy: *H. Yoshino*, Justice and Logic. They relevant paper was published as: The Role of Deductive Methods in Reasoning about Justice, in: Justice. The Annual of Legal Philosophy (1974), pp. 38-68.

cal decision.

It is sometimes said that a juridical decision is not obtained by logical deduction, but rather by induction on each juridical experience and based on a socio-economical basis. This assertion is partly true and partly false. In my opinion, the inference rule of "modus tollens" as a hypothetical-deductive method for testing by falsification is basic even in the framework of "inductive reasoning" starting from particular legal facts.

4.2 The Logical Structure of Argumentation in Juridical Decision as "Modus Tollens"

Starting from a particular juridical experience, lawyers formulate a general legal-normative statement, (N_1), or a legal-dogmatic theory, (N_1), which contains the statement as a provisional hypothesis. Hence, juridical experience is compared to certain statements, such as the code of law, statements of jurisdiction and various assumptions which correspond to the general sense of justice. Lawyers test the soundness of this juridical-normative statement against the particular juridical-normative statement ($N_{1,1}$, $N_{1,2}$, $N_{1,3}$, ..., $N_{1,n}$), which is deducible from the general one. If a particular juridical-normative statement is negated ($\neg N_{1,n}$), then the general juridical-normative statement in question is also negated ($\neg N_1$). The logical structure of this form of argument is as follows:

(3) $$(N_1 \rightarrow N_{1,n}) \wedge \neg N_{1,n} \rightarrow \neg N_1$$

This formula has the following reading: "If N_1 is true then $N_{1,n}$ is true, but if $N_{1,n}$ is false then the falsity of N_1 is derivable from the conjunction of premises."

This formula is valid since it is an instance of "modus tollens." The validity of this inference can also be confirmed by applying the shortcut truth-table method; the assignment of truth-values yields a

contradiction.[29]

$$\begin{array}{ccccccccc}
(N_1 & \rightarrow & N_{1,n}) & \wedge & \neg & N_{1,n} & \rightarrow & \neg & N_1 \\
+ & + & - & + & + & - & - & - & + \\
4 & 3 & 5 & 2 & 3 & 4 & 1 & 2 & 3 \\
\uparrow & \uparrow & \uparrow & & & & & &
\end{array}$$

Note, however, the following two points: First, in this formalization the truth-values "1" and "0" are read normatively as "correct" or "incorrect." Therefore, the negation (the negative normative assignment) of the more general and the particular juridical-normative statements are assigned the truth-value "0" (false), while the positive assignments have the truth-value "1" (true). Second, all propositional letters in this formula express normative statements in such a way that the assignment of normatively read truth-values can be done uniformly. In light of the two points mentioned above, the falsification scheme of argumentation in juridical decision making can be adequately formulated. The former schemes in Section 2 [(A)‴ and (B)‴] offer a semantic foundation for this formalism.[30]

If the particular juridical-normative statement is not negated ($N_{1,n}$), the general juridical-normative statement in question is temporarily corroborated. The logical structure of the argument is as follows:

(4) $$(N_1 \rightarrow N_{1,n}) \wedge N_{1,n} \rightarrow N_1$$

This inference is not logically valid; the application of the shortcut truth-table method yields no contradiction in the assignment of truth-values.

29 For a description of the shortcut truth-table method, see e.g. *I. Tammelo, H. Schreiner*, Grundzüge und Grundverfahren der Rechtslogik, vol. 1, Pullach near Munich 1974, pp. 30 ff.

30 Cf. Section 2 of this contribution.

$$
\begin{array}{ccccccc}
(N_1 & \rightarrow & N_{1,n}) & \wedge & N_{1,n} & \rightarrow & N_1 \\
- & + & + & + & + & - & - \\
4 & 3 & 4 & 2 & 3 & 1 & 2
\end{array}
$$

Thus, the more general juridical-normative statement is not proven, but temporarily corroborated. One cannot preclude the possibility that another particular juridical-normative statement $(N_{1,n+1})$, which is also deducible from the general statement, is evaluated negatively (falsified) and so the general statement is also evaluated negatively (falsified).

(5) $$(N_1 \rightarrow N_{1,n+1}) \wedge \neg N_{1,n+1} \rightarrow \neg N_1$$

Consequently, I am convinced that in juridical argumentation, decisions cannot and should not be seen as being based on verification, but rather on falsification of the general juridical-normative statement.[31,32] In this way, the temporarily settled general juridical-normative statements are examined by several particular juridical-normative statements which are deducible from the former ones. This is done when an (important) falsification is encountered. A (general) juridical-normative statement is accepted, if it is sufficiently tested and not falsified; hence it is asserted that the juridical-normative statement is corroborated, thus, relatively correct. Eventually, it becomes accepted as a juridical decision.

31 This is a starting point for inferring the problematic nature of the term "external justification" (cf. footnote 46 of this paper).

32 How is the falsification of a particular juridical-normative statement effected? I believe that the inference rule of "modus tollens" is essential here too, as long as falsification can be found. The most elementary (juridical-normative) statement, one that cannot be deduced by some reasoning process, has to be accepted or rejected as a subjective value judgement. These relations also hold for the effect evaluation as "modus tollens in the broad sense," to be explained in the next section.

The whole process of argumentation of juridical decision making yielding a (temporarily) corroborated juridical-normative statement can be sketched as follows;

(6) 1. $(N_1 \rightarrow N_{1,1}) \wedge N_{1,1}$
$(N_1 \rightarrow N_{1,2}) \wedge N_{1,2}$
....
....
$(N_1 \rightarrow N_{1,n}) \wedge \neg N_{1,n} \rightarrow \neg N_1$: N_1 is falsified (evaluated normatively-negative)

(7) 2. $(N_2 \rightarrow N_{2,1}) \wedge N_{2,1}$
$(N_2 \rightarrow N_{2,2}) \wedge N_{2,2}$
....
....
$(N_2 \rightarrow N_{2,n}) \wedge \neg N_{2,n} \rightarrow \neg N_2$: N_2 is falsified (evaluated normatively-negative)

$$
(8)\quad n.\quad \left.\begin{array}{l}
(N_n \rightarrow N_{n,1}) \wedge N_{n,1} \\
(N_n \rightarrow N_{n,2}) \wedge N_{n,2} \\
\cdots\cdots\cdots\cdots \\
\cdots\cdots\cdots\cdots \\
(N_n \rightarrow N_{n,m}) \wedge N_{n,m}
\end{array}\right\} \cdots\cdots N_n:
$$

N_n is corroborated (N_n is accepted as the result of a juridical decision)

In this way, more general juridical-normative statements are tested by particular juridical experience in a bottom-up fashion, so to speak. This is done systematically by logical reasoning (in the direction of induction); namely, either it is falsified or corroborated. In this sense, argumentation in juridical decision making has the logical structure of the inference rule "modus tollens."[33]

33 This rule of inference ("modus tollens") and the scheme outlined of reasoning above are valid for modifications of juridical decision over time (cf., footnote 42 in this paper).

4.3 The Logical Structure of Argumentation in Juridical Decision as "Modus Tollens in the Broad Sense"

In Section 4.2, I described juridical decision-making as falsification-based by means of the inference rule "modus tollens." In this kind of falsification, the particular juridical-normative statement is deducible from the more general juridical-normative statement, whereby the latter is falsified if the former is falsified. Falsification of a juridical statement has another aspect of reasoning, which is not "modus tollens" in the exact classical sense of logic, but very similar. I will call this form of reasoning "modus tollens in the broad sense." This kind of reasoning applies mainly to falsification as performed by effect evaluation.[34]

The logical structure of argumentation in juridical decision-making by means of effect evaluation is as follows: If a juridical-normative statement or a juridical-normative theory (Na_1) is accepted, the effects ($E_{1,1}$, $E_{1,2}$, $E_{1,3}$, ..., $E_{1,n}$) follow as a consequence of the application of such a juridical-normative statement. Some of these results are evaluated negatively. Hence, one can conclude that the original juridical-normative statement has to be evaluated negatively. This line of reasoning has a structure similar to falsification by means of "modus tollens." The logical structure of this inference can be represented as follows:

$$(9)\quad (Na_1 \Rightarrow E_{1,1}) \wedge (Na_1 \Rightarrow E_{1,2}) \wedge \ldots \wedge (Na_1 \Rightarrow E_{1,n}) \wedge -E_{1,n} \rightarrow -Na_1$$

Falsification here concerns the subformula:

$$(10)\qquad (Na_1 \Rightarrow E_{1,n}) \wedge -E_{1,n} \rightarrow -Na_1$$

The reading of this formula is: "From the acceptation of the juridical-

34 For a discussion of effect evaluation in legal science, cf., *A. Podlech*, pp. 185-223, especially p. 201.

normative statement (Na_1) the effect ($E_{1,n}$) follows as a result of the application of the statement. Since the effect is evaluated normatively-negative ($-E_{1,n}$), it follows that the acceptance of the juridical-normative statement is evaluated normatively-negative ($-Na_1$)." This inference, is close in its effect to "modus tollens." Below, I will call such an inference rule "modus tollens in the broad sense."

The entire process of argumentation of juridical decision as performed by "modus tollens in the broad sense," which leads to a (temporarily) corroborated juridical-normative statement, could be described in the same way as above (6)–(8) by means of "modus tollens" in the proper sense, but this is not done here.[35]

In the formalizations (9) and (10) above, the special operators "$\Rightarrow$" and "$-$" are used to make this kind of reasoning more transparent.[36] Our formalization deviates from the formalization of classical (mathematical) logic. In order to prove that the inference "modus tollens in the broad sense" is valid and in order to develop a logical calculus for the application of classical (mathematical) logic to juridical argumentation, we need to transform the (extended) schemes (9) and (10) to classical (mathematical) logic.[37] This will be done below.

This transformation requires consideration of two problems. On one the hand, there is the problem of formalizing the falsification, i. e., the normatively negative evaluation of results. The propositional letter "$E_{1,n}$" in the first conjunct of the antecedent should be evaluated as

35 A sketch of this form of argument would correspond to the schemes (6)–(8) in this paper.

36 These operators are introduced for only one reason: the usage of these operators shows the logical structure of the argumentation we are concerned with; they are simpler and therefore easier to understand. In order to develop a logical calculus, we would need to extend the formation rules and give a fixed definition of these operators, in the sense of a semantical justification. I do not want to deal with this problem here, as it is not essential in enhancing the readability of the formulae.

37 See Section 2 in this paper.

indicatively true, while the same letter in the second conjunct (of the antecedent) should be evaluated as normatively false. Within logic, evaluating the same logical symbol as indicatively true or false in one instance, and normatively true or false in another is not permissible. Being aware of this problem, I used different symbols in the formulas (9)−(10) and (3)−(5). In the latter formulae, each propositional letter can be uniformely evaluated by normative truth such that no problems occur with negation.

On the other hand, there is the problem of how to formalize the effect of applying a norm. Hence, I used different symbols for implication in the formulae (9)−(10) and (3)−(5).[38] In order to correctly formalize the effect of applying a norm, we would have to solve the problem of formalizing causality.

When giving a precise formalization of "modus tollens in the broad sense," I do not preclude the possibility of formalizing the logical structure of juridical falsification by effect evaluation utilizing the inference rule "modus tollens" in a precise way. This could be done by introducing additional formation and transformation rules. However, I can see the possibility of reformulating the formalization to classical(mathematical) logic by adding a universal assumption, which is peculiar to juridical practice and captures the meaning of "modus tollens in the broad sense." By way of example, below I attempt such a reformulation along the lines of the aforementioned argument shown just.

I am convinced that the following assumption is tacitly accepted in juridical argumentation:

(11) "If the application of a juridical-normative statement (legal norm

38 It is possible to express the relation of logical consequence by means of (material) implication which is used in the formalizations (3)−(8). It would be also possible to represent the description of the effect relation (which is mainly a relation of causality) by means of (material) implication. In this case, however, special conditions for such an implication are needed.

and juridical theory included) results in an effect which is to be evaluated negatively, then the juridical-normative statement is also to be evaluated negatively."

This assumption should not be considered to be a logical rule, but a premise in logical reasoning. If this assumption is added as a supplementary assumption, then juridical falsification by effect evaluation, i. e. "modus tollens in the broad sense," can be reconstructed as a classical, logically valid inference as described below:

The following letters are used in the formulae:

$N(.)$. is a juridical-normative statement (legal norm and juridical theory included)
$P(.)$. is a phenomenon
$Rs(., ..)$ application of . results in ..
$Ne(.)$. should be evaluated negatively[39]

(12) $\forall n \forall s\ (N(n) \wedge P(s) \wedge Rs(n,s) \rightarrow ((P(s) \rightarrow Ne(s)) \rightarrow Ne(n)))$
(13) $N(n_1) \wedge P(s_1) \wedge Rs(n_1,s_1)$
(14) $\underline{P(s_1) \wedge Ne(s_1)}$.
(15) $Ne(n_1)$

This inference can be rewritten as the following formula:

(16) $(\forall n \forall s\,(N(n) \wedge P(s) \wedge Rs(n,s) \rightarrow ((P(s) \rightarrow Ne(s)) \rightarrow Ne(n)))) \wedge$
$\wedge\ (N(n_1) \wedge P(s_1) \wedge Rs(n_1, s_1)) \wedge\ (P(S_1) \wedge Ne(s_1)) \rightarrow Ne(n_1)$

The (implicational) formula (16)−with (12)−(14) as antecedents and (15) as conclusion−is logically valid, which (12) encodes assumption (11).

Logical validity can be proven as follows:

1. $\forall n\ \forall s(N(n) \wedge P(s) \wedge Rs(n,s) \rightarrow ((P(s) \rightarrow Ne(s)) \rightarrow Ne(n)))$

39 Note that the "obligatory aspect" of statement (11) is covered by the predicate (cf., formalization (1)).

2. $N(n_1) \wedge P(s_1) \wedge Rs(n_1,s_1)$
3. $P(s_1) \wedge Ne(s_1)$ $\quad$ / $\underline{Ne(n^1)}$
4. $\forall s(N(n_1) \wedge P(s) \wedge Rs(n_1,s) \rightarrow ((P(s) \rightarrow Ne(s)) \rightarrow Ne(n_1)))$ 1., U.I.
5. $N(n_1) \wedge P(s_1) \wedge Rs(n_1,s_1) \rightarrow ((P(s_1) \rightarrow Ne(s_1)) \rightarrow Ne(n_1))$ 4., U.I.
6. $(P(s_1) \rightarrow Ne(s_1)) \rightarrow Ne(n_1)$ 5., 2., M.P.
7. $P(s_1) \wedge Ne(s_1) \rightarrow Ne(s_1)$ Axiom
8. $Ne(s_1)$ 3., 7., M.P.
9. $Ne(s_1) \rightarrow (P(s_1) \rightarrow Ne(s_1))$ Axiom
10. $P(s_1) \rightarrow Ne(s_1)$ 8.,9., M.P.
11. $Ne(n_1)$ 6., 10., M.P.

In this way, the logical structure of juridical falsification by "modus tollens in the broad sense" can be reconstructed within classical (mathematical) logic. But I do not assert that this reformulation is the best solution; it is only one possible solution. I will seek a better solution in a future publication.

Finally, it should be stressed again that the inference rules of "modus tollens" mentioned above, i.e., "modus tollens" proper, and "modus tollens in the broad sense" are basic schemes of juridical argumentation in legal decision-making. Nowadays, it is the prevailing opinion in both the Federal Republic of Germany and Japan that juridical decision-making is not direct deduction from statutes, but is obtained between statutes and individual circumstances by the "constant interaction, a to-and-fro movement of one's focus" (*K. Engisch*)[40] or "bringing-to-accordance" (*Arthur Kaufmann*).[41] This relation of "constant interaction, the to-and-fro movement of one's focus" can be considered to be a logical structure, i. e., "modus tollens" in the proper or in the broad sense. This inference framework is also valid for the line of reasoning in case a construction of statute is modified; moreover, it is valid for new legislation resulting from a change in the socio-economic situation.[42]

40 *K. Engisch*, Logische Studien zur Gesetzesanwendung, Heidelberg 1942, second edition 1960, p. 15.

41 *Arthur Kaufmann*, Analogie und "Natur der Sache." Zugleich ein Beitrag zur Lehre vom Typus, Karlsruhe 1965, p. 29; also published in: A. Kaufmann, Rechtsphilosophie im Wandel. Stationen eines Weges, Frankfurt/Main 1972, pp. 272-320, p. 302.

42 I put that up for discussion at a conference on the historical method in legal science in Turku, Finland (13. 12. 1979), which was supervised by Professor H. T. Klami. I argued (and still argue) as follows:

A juridical-normative statement, including legal norm and juridical theory (N_1), is evaluated normatively positive (correct) – is corroborated – at time point 1 ($T.1$). However, the socio-economic situation changes and so does public opinion. As a consequence, the statement is evaluated normatively negative (incorrect) – falsified – at time point 2 ($T.2$). Thereby, effect evaluation is considered in both cases. The logical structure of this line of reasoning can be sketched as follows:

(a) $T.1.: (N_1 \Rightarrow E_{1,1}) \wedge E_{1,1} \ldots\ldots N_1 : N_1$ is corroborated
(b) $T.2.: (N_1 \Rightarrow E_{1,1}) \wedge -E_{1,1} \rightarrow -N_1 : N_1$ is falsified
(c) $T.2.: (N_1 \Rightarrow E_{1,2}) \wedge -E_{1,2} \rightarrow -N_1 : N_1$ is falsified
(d) $T.2.: (N_2 \Rightarrow E_{2,n}) \wedge E_{2,n} \ldots\ldots N_2 : N_2$ is corroborated

As shown above, the logical structure of decision due to a modification in a construction of a statute or a new legislation is the inference rule of "modus tollens." (b) shows: as in (a), application of the juridical-normative statement N_1 yields the effect $E_{1,1}$, but unlike (a), at time point 2 public opinion has changed; more precisely, the criterion of evaluation has changed such that the effect is evaluated negatively ($-E_{1,1}$), hence N_1 is evaluated negatively ($-N_1$). (c) shows: at time point 2 there is another effect $E_{1,2}$ due to a change in the socio-economic situation, which – after applying N_1 – is evaluated negatively ($-E_{1,2}$); it follows that N_1 is evaluated negatively. Therefore, at time-point 2, the juridical-normative statement N_1 is abandoned and another candidate for juridical-normative statements N_2 is given. (d) shows: the statement is corroborated through the effect evaluation. Concerning the corroboration in (a) and (d), it is not sufficient to test once; one has to do several examinations (cf. *the formalization (8) in this* document).

5 Logical Analysis of Argumentation in Juridical Decision

In my opinion, several decisions involve argumentations that have the logical structure of "modus tollens" or "modus tollens in the broad sense." This has already been demonstrated in a logical analysis of Japanese jurisdiction. Subsequently, I want to give an example of jurisdiction from Germany, namely the *Bundesgerichtshof* (German Federal Court of Justice) ; thereby, I will analyze the logical structure in order to prove, for instance, that juridical decision has the logical structure of "modus tollens in the broad sense."

BGHSt 25, 30.[43] This jurisdiction is concerned with the element of "using signs" within the meaning of § 86a (1) of the German Criminal Code.

The relevant law says:

> § 86a The German Criminal Code:[44]
> "Use of signs of anti-constitutional organizations:
> (1) Whoever propagates in public signs of one of the parties or associations specified in § 86 (1), (2), and (4) within the spatial purview of this law, shall be punished with up to three years of imprisonment or penalty."

In the reasons of this judgement, wherein the facts are of no importance for this logical analysis, the following argumentation can be found (the labels are mine, H.Y.):

"§ 86a Criminal Code does not contain as an element of the offense a concrete endangering... If the legislator did not require a concrete endangering in § 86a Criminal Code, then the interpretation considered by the Federal Attorney General, referring to Schröder, that the use of such a sign only constitutes the offense if the circumstances of the use suggest an endangering cannot be agreed with.

(17) *Such an interpretation* (Na_1) would lead, in its practical application, to *results similar to those required in a concrete endangering* (E_1).

(18) *It* (Na_1) would furthermore *in practice lead to unusually great difficulties and uncertainties in the subsumption*($E_{1.1}$) and would result *in many cases in difficulties of evidence not exactly as great, but still similar* ($E_{1.2}$),

(19) like they just had to be avoided, according to the Select Committee for Penal Reform.

(20) *With it* (Na_1) too, *the realization of the purpose of the law, which has deliberately been chosen to be so wide, would be jeopardized* (E_2)."

43 BGHSt = Entscheidungen des BGH in Strafsachen (Decisions of the Federal Court of Justice in Criminal Cases).

44 § 86a (1) has been reformulated by an act of parliament on October 28, 1994 (BGBl. I, 3186).

Now I want to formalize (in propositional logic) these arguments by means of the letters given above. The arguments (17), (18) and (19) can be provisionally formalized as follows:

(17′) $Na_1 \Rightarrow E_1$

(18′) $(Na_1 \Rightarrow E_{1.1}) \land (Na_1 \Rightarrow E_{1.2})$

(20′) $Na_1 \Rightarrow E_2$

In the statements of the judgement cited above, the negative evaluation of the effects is not made explicit. However, it is obvious from the context that the effects are evaluated negatively here. This can be seen from the fact that the argumentation uses the subjunctive and from statement (19), where the court demonstrates the negative evaluation of the effects, following the Select Committee on Penal Reform. If one tries to formalize the above argumentation precisely—thereby considering the subjunctive and the partly implicitly given negative evaluations—then the following formulae result:

(17″) $(Na_1 \Rightarrow E_1) \wedge - E_1$
(18″) $((Na_1 \Rightarrow E_{1.1}) \wedge (Na_1 \Rightarrow E_{1.2})) \wedge (- E_{1.1} \wedge - E_{1.2})$
(20″) $(Na_1 \Rightarrow E_2) \wedge - E_2$

It is obvious from the reasons given above that the construction of the statute leading to these effects is evaluated negatively even in the judgement, as a consequence of the argument. The logical formula corresponding to the negative evaluation of the construction of the statute under consideration is:

(21″) $- Na_1$

Hence the above argumentation can be formalized as a whole as follows:

(17″–21″) $((Na_1 \Rightarrow E_1) \wedge - E_1) \wedge (((Na_1 \Rightarrow E_{1.1}) \wedge (Na_1 \Rightarrow E_{1.2})) \wedge$
$\wedge (- E_{1.1} \wedge - E_{1.2})) \wedge (Na_1 \Rightarrow E_2) \wedge - E_2) \rightarrow - Na_1$

This formula typically has the logical structure of "modus tollens in the broad sense." This formula can be decomposed into elementary formulae, each of which contains a "modus tollens in the broad sense."[45]

(17‴) $(Na_1 \Rightarrow E_1) \wedge - E_1 \rightarrow - Na_1$
(18a‴) $((Na_1 \Rightarrow E_{1.1}) \wedge - E_{1.1}) \rightarrow - Na_1$
(18b‴) $((Na_1 \Rightarrow E_{1.2}) \wedge - E_{1.2} \rightarrow - Na_1$
(20‴) $(Na_1 \Rightarrow E_2) \wedge - E_2) \rightarrow - Na_1$

In the present judgement, all other arguments have the logical structure of "modus tollens in the broad sense," too. For instance, in the next argument, a different construction is falsified by an analogous effect evaluation, which also has the logical structure of "modus tollens in the broad sense." Due to space limitations, the logical formalization of that argumentation is not demonstrated here.

6 Conclusion

In this paper, I have aimed to clarify the logical structure of argumentation in juridical decisions. The theses for which I argued and which are partly proven can be succinctly summarized as follows:

1. Justification is a question of logical consequence. This is not only valid for so-called *internal justification*, but for so-called *external justification* as well. In this respect, there is no essential difference between the two.[46]

45 As seen in the analysis of the reasons in the present judgement, the effect evaluation is not performed only once but several times. From the logical point of view a *single* falsification is sufficient, especially with proper "modus tollens." In the reasons of a judgement it is recommendable to present the falsification of more effects of the application (of a juridical-normative statement) for two reasons: on one hand, falsification (normatively negative evaluation) of the effect of a juridical-normative statement is not intersubjective as in natural science but more subjective; on the other hand, the relation between the application of the juridical-normative statement and its result (effect) cannot be stated as precisely as in natural science. If the falsification of more effects is given, other persons can be better convinced by the juridical decision under consideration. Hence, the subjectivity of a juridical decision is reduced.

46 In my opinion, it would not be adequate then to use the term "justification" for argumentation in the sense of "external justification" because in this kind of argumentation decision is done, that is, the acception of the juridical-normative statement, and this decision is not finitely (logically) justifiable; moreover, it cannot be "verified" but only "falsified".

2. The justification of the ultimately elementary stated assumption in juridical decision cannot be done by deduction from other statements, but must be determined (or decided).
3. In order to analyze the logical structure of juridical argumentation, the method of classical (mathematical) logic can be adequately applied.
4. To make the logical structure of juridical decisions clear, Popper's falsifiability thesis should be introduced to juridical argumentation.
5. I propose the scheme of falsification by "modus tollens" as a basic scheme of argumentation in juridical decision.
6. On one hand, the logical structure of a juridical decision can be captured in an exact logical sense, by "modus tollens" in the proper sense, where a particular juridical-normative statement is deduced from a more general juridical-normative statement (including a legal norm or juridical theory). On the other hand, the logical structure of a juridical decision can be captured in another sense, as "modus tollens in the broad sense," whereby effect evaluation of the application of a juridical-normative statement are taken to considered. In both cases, we presented a formalization of the respective argumentations.
7. The logical structure of a juridical decision as "modus tollens in the broad sense" has been proven by way of example of an analysis of a judgement of the German Federal Court of Justice.
8. As a result of the above, analyses and proofs, it can be finally argued that one should direct one's attention more to the problem of falsification and falsifiability in juridical argumentation (as opposed to external justification and external justifiability).