

The Logical Structure of a Legal System Proving the Validity of Law

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This paper clarifies the logical structure of a legal system in terms of the validity of legal sentences. The law must be legally valid in order that it can be applied to a concrete case. Legal rules which are applied to a case are to be proved as legally valid. How can this proof be done in law? In other words, what is the structure of legal system which enables the proof? The validity of a legal sentence can be proven if it is deduced from other valid legal sentences. Therefore, the logical structure of a legal system that makes such a deduction possible is clarified in this paper.

Inhaltsverzeichnis

1. Introduction
2. Structure of Legal Sentences as an Element of a Legal System
 - 2.1. Legal rule and fact sentences
 - 2.2. Legal elementary and complex sentences
 - 2.3. Legal object and meta-sentences
3. Structure of Connection of Legal Sentences for a Legal System
 - 3.1. «AND» structure of the connection of legal sentences
 - 3.2. Connection of legal sentence in complex sentence
 - 3.3. Connection of object sentence with meta-sentence
 - 3.4. Connection of a legal meta-sentence with another meta-sentence
4. Logical Structure of System of Law as a Whole
 - 4.1. Starting point and strategy: Systematization through legal reasoning
 - 4.2. Inference to decide legal relations
 - 4.3. Legal meta-inference
 - 4.4. The most fundamental legal meta-rule sentence regulating that a legal sentence is valid
 - 4.5. Fundamental legal meta-rule sentences regulating that a legal sentence becomes valid.
 - 4.6. Positive legal meta-rule sentences assisting fundamental legal meta-rule sentences

- 4.6.1. Positive legal meta-rule sentences regulating the formation of legal sentences
- 4.7. The validity of legal meta-rule sentences
 - 4.7.1. The validity of fundamental legal meta-rule sentences
 - 4.7.2. The validity of positive legal meta-rule sentences
 - 4.7.3. The level of the validity of positive legal meta-sentences
 - 4.7.4. The basic legal meta-sentences

5. Conclusion

1. Introduction ^

[Rz 1] The systematization of law has been an important target of theories of law. Modern natural law theorists like Samuel von Pufendorf have tried to construct a deductive system of law¹. Legal positivist Hans Kelsen has developed a legal positivistic theory systematizing the hierarchical structure of a legal norm system. The systematization by modern natural law theories is not logically correct. Kelsen's theory² is interesting, but fails to utilize precise logical methods. We would like to clarify how the systematization of law can be explained with precise logical methods in terms of Logical Jurisprudence.³

[Rz 2] Logical Jurisprudence is a logical theory of law which was developed by the author based on constructing legal reasoning systems or legal knowledge base systems. Logical Jurisprudence analyzes languages in law logically and clarifies the logical structure of thought in law. Similar to physics, Logical Jurisprudence explains the whole legal system from minimum elements. It starts from three primitives: «sentence», «validity» of a sentence, and «inference rule». Starting from these three primitives, in this paper the author analyzes how the validity of legal sentences is to be inferred to construct a legal system as a whole.

2. Structure of Legal Sentences as an Element of a Legal System ^

[Rz 3] A legal system is to be analyzed and constructed in terms of three types of alternative fundamental concepts of legal sentence: (1) legal rule sentence and fact sentence, (2) legal element sentence and complex sentence and (3) legal object sentence and meta-sentence. These are explained below.

2.1. Legal rule and fact sentences ^

[Rz 4] **Legal rule sentences** have the following syntactic structure:
 $a(X) \leftarrow b(X)$.

[Rz 5] This formula is read: «For all X , if X is b , then X is a .»⁴ The consequence of the sentence, which is the formula at left in the implication « \leftarrow », is called a «legal consequence» and the antecedent, which is the formula at right, is called a «legal requirement». For example, «If an

offer reaches an offeree, then the offer becomes effective».

[Rz 6] **Legal fact sentences** have the following syntactic form:

$b(x1).$

[Rz 7] This formula is read: « $x1$ is b ». For example: «*Anzai's offer reached an offeree*». It is to be noted that the difference between the legal rule sentence and fact sentences is purely abstract in Logical Jurisprudence. Logical Jurisprudence does not agree with methodological dualism in terms of norm and fact or «ought» and «is», as proposed by Kelsen.

2.2. Legal elementary and complex sentences ^

[Rz 8] **An elementary sentence** is the smallest unit of legal sentences. Statutes or contracts are composed of elementary legal sentences (e.g., «*One must drive a car under 100 km /hour on a highway.*»)

[Rz 9] A **complex legal sentence** is a group of legal sentences which has a unique name. For example:

The United Nations Convention on Contracts for the International Sale of Goods;

Contract for Sale of a Farming Machine between Anzai and Bernard on October 8, 2010.

[Rz 10] Not only a code, but also parts, chapters, sections, or an article of a code can be a complex legal sentence. In most cases, the fact that a certain legal sentence belongs to a complex legal sentence is represented by the place and the space where they are printed. The relationship is represented in Logical Jurisprudence by a sentence describing the united relationship of grouped sentences. If sentence identifier $cs1$ is used for a complex sentence and $es1$, $es2$ and $es3$ are used for elementary sentences, then the relation of a complex sentence with its element sentences are formalized as below:

$complex_sentence(cs1,[es1,es2,es3]).$

2.3. Legal object and meta-sentences ^

[Rz 11] A **legal object sentence** describes the object itself. In the legal domain, the object is an «obligation». Legal object sentences describe the obligations of a person. Sentences such as, «one must drive a car under 100 km/hour on a highway» and «it is obligatory for B to pay A the price of \$10000», are legal object sentences. The obligation is described in object sentences using a normative predicate such as, «must», «ought», «obligatory» and so on.

[Rz 12] A **legal meta-sentence** describes a legal sentence. More precisely, it describes the validity of a legal sentence. An example of a legal meta-sentence is: «(1) This Convention applies to

contracts of the sale of goods between parties whose places of business are in different states: (a) when states are contracting states; or ... » (Article 1 of the CISG⁵). There is also a legal meta-sentence which describes the validity of another legal meta-sentence.

3. Structure of Connection of Legal Sentences for a Legal System ^

3.1. «AND» structure of the connection of legal sentences ^

[Rz 13] The logical word, «AND» (&), is used to connect plural legal sentences in a legal system, if they are asserted as true or valid in a reasoning word.

3.2. Connection of legal sentence in complex sentence ^

[Rz 14] A legal complex sentence is constituted by plural legal sentences which are connected with the logical word «AND». Such a plural legal sentence can also be a legal complex sentence. The final unit of member of the complex sentence is a legal elementary sentence. The concept of a complex legal sentence enables us to simultaneously evaluate the validity of legal sentences.

3.3. Connection of object sentence with meta-sentence ^

[Rz 15] People's conduct is ultimately regulated by legal obligations when law is applied to maintain the social order. What does it mean that people commonly perceive such a legal obligation to exist? Logical Jurisprudence considers: such a perception is based on the legal fact that the legal object sentence, which describes the relevant obligation, is valid. This is described with a legal meta-sentence. For example:

« *It is obligatory for Anzai that Anzai deliver the goods to Bernard* is valid on 15. april 2010».

[Rz 16] What is the logical structure of the connection of a legal object sentence with a legal meta-sentence? In other words, how is this connection to be formalized logically? The above example is to be formalized as follows:

s1: is_obligatory(⟨Anza⟩, deliver(⟨Anzai⟩, ⟨Bernar⟩, goods(agm151))) .

*s2: is_valid(s1, t04_15_2010)*⁶ .

[Rz 17] Here *s1* stands for an object sentence and *s2* a legal meta-sentence. *s1* and *s2* are identifiers for each legal sentence. It is to be noted that a legal sentence is formalized as an entity which has a unique name.

3.4. Connection of a legal meta-sentence with another meta-sentence ^

[Rz 18] A legal meta-sentence, which describes that a legal object sentence is valid, is to be proven true in order that the validness of the object sentence is guaranteed. For such proof, one must deal with the validity of a legal meta-sentence. For instance, the validity of the legal meta-sentence at time April 15th, 2010 is formalized by another legal meta-sentence³ as follows:

s3: is_valid(s2,t15_04_2010).

4. Logical Structure of System of Law as a Whole ^

4.1. Starting point and strategy: Systematization through legal reasoning ^

[Rz 19] The system of law is created through legal reasoning. One could say, «If there is no legal reasoning, then there is no legal system.» It is necessary for us to analyze the process of legal reasoning to clarify how the legal system is created through legal reasoning. Only valid legal sentences are true in the legal world and can be applied to solve problems. The law that is applicable to solve a legal problem at a certain time and place is the whole of the legal sentences which can be proven as valid. The system of law is therefore the system according to which legal sentences to solve problems are proven as valid. It is therefore necessary for us to clarify how the validity of legal sentences is to be proven through legal reasoning.

4.2. Inference to decide legal relations ^

[Rz 20] The goal of legal reasoning in the substantial law field is to decide legal relations between parties. Legal relations are composed of legal rights and obligations. To decide what kind of legal rights and obligations exist is to infer what kind of legal sentences describing such rights and obligations are valid. This must be done through proving legal meta-sentences which describe the validity of a legal sentence describing such rights and obligations.

[Rz 21] A legal sentence which describes one's right is, however, not a legal object sentence but a legal meta-sentence, because it is a legal sentence to afford him a legal power to settle a legal sentence. Therefore, the final target of legal reasoning is to decide what kind of legal object sentences describing obligation are valid. The objective legal state of affairs is represented by legal object sentences whose validity is proven. The legal meta-sentences which describe the validity of object sentences are to be proven through legal reasoning. The whole legal object sentences, the validity of which is proven, represent the whole legal obligations (i.e., the whole objects of the relevant legal world).

4.3. Legal meta-inference ^

- [Rz 22] Legal rule sentences which are applied to solve a problem must be valid. In other words, only valid legal rule sentences are to be applied to solve a problem. For a legal inference, it is necessary to infer to prove that legal rule sentences applied are valid. An inference to decide the validity of the legal rule sentence applied is called as legal meta-inference. Legal meta-inference controls legal reasoning by deciding the validity of legal rule sentences which are to be applied in a legal inference.
- [Rz 23] In legal meta-inference, legal meta-rule sentences are applied to decide the validity of a legal sentence. If a goal is proven as true in an inference through the application of a legal rule sentence, the meta-inference must start to solve another goal, whether the legal rule sentence applied is valid. If the second goal is proven as true, then the proof of the first goal is to be accepted as just, in other words, the first inference is admitted as correct.⁷
- [Rz 24] We would like to clarify what kind of legal meta-rule sentences are to be applied in legal meta-inference to construct a legal system. Logical Jurisprudence has found several fundamental legal meta-rule sentences, which decide the validity of legal sentences, as implicit legal common sense through the study on constructing legal knowledge base. Some of legal meta-rules are explained in the next sections.

4.4. The most fundamental legal meta-rule sentence regulating that a legal sentence is valid ^

- [Rz 25] The following fundamental legal meta-rule sentence is to be valid to decide that a legal sentence is valid.
- $$r0: is_valid(S, T) \leftarrow (become_valid(S, T1) \ \& \ before(T1, T)) \ \& \ not(is_terminated(S, T2) \ \& \ before(T2, T)).$$
- [Rz 26] This rule sentence is to be read: *if a legal sentence S becomes valid at time T1 before time T and it is not the case that S is terminated at time T2 before T, then S is valid at T.*
- [Rz 27] This legal meta-rule sentence is always applied whenever it is decided whether a legal sentence is valid. This meta-rule sentence is considered as the most fundamental legal meta-rule sentence⁸. This legal rule sentence cannot be found as a statutory text in any legal regulation. This is implicitly taken for granted by all regulations. Without this rule, no statutory legal sentence works, when it comes to application. This rule enables us to put a mere collection of legal sentences into a legal system. All other legal meta-rule sentences regulate the fulfillment of the first requirement (S becomes valid) or the second requirement (S is terminated) of this rule.

4.5. Fundamental legal meta-rule sentences regulating that a legal sentence becomes valid. ^

[Rz 28] There are several fundamental legal meta-rule sentences which decide the first requirement of the most fundamental meta-rule sentence $r0$. In terms of the relation of an elementary sentence with a complex sentence, the following fundamental legal rule sentence is to be valid:

$r01: \text{become_valid}(ES, T) \leftarrow$
 $\text{element_complex_sentence}(ES, CS) \ \& \ \text{become_valid}(CS, T).$

[Rz 29] This rule is to be read: *if ES is an element sentence of a complex sentence CS and CS becomes valid at time T, then ES also becomes valid at T.*

[Rz 30] In order that a complex sentence becomes valid, the following legal meta-rule sentence is to be conceived:

$3AA1: \text{become_valid}(\text{complex_sentence}(CS, ES), T) \leftarrow$
 $\text{is_formed}(\text{complex_sentence}(CS, ES), T1) \ \&$
 $\text{not}(\text{invalid}(\text{complex_sentence}(CS, ES), T)) \ \&$
 $((\text{beginning_time_of_effectiveness}(TE, CS, T)$ or
 $\text{condition_of_effectiveness}(CE, CS, T)) \text{ or } T=T1)^9.$

[Rz 31] This rule is to be read: *A complex sentence CS becomes valid at time T if the complex sentence CS is formed at time T1 and it is not the case that the complex sentence is invalid at time T, and ((the complex sentence entails the beginning time of effectiveness and the beginning time has come at T) or (the complex sentence entails a condition of the effectiveness and the condition is fulfilled at time T)) or T is T1.*

[Rz 32] In a legal domain the following legal meta-rule sentence must then also be valid:

$3AA2: \text{become_valid}(\text{obligatory}(X, Z), T) \leftarrow$
 $\text{require}(Y, X, Z, T) \ \& \ \text{is_valid}(\text{may}(Y, X, \text{require}(Y, X, Z, T)), T).$

[Rz 33] This rule is to be read: *if Y requires X to do Z at time T and «Y may require X to do Z at T» is valid at T, then «it is obligatory for X to do» becomes valid at T.*

4.6. Positive legal meta-rule sentences assisting fundamental legal meta-rule sentences ^

[Rz 34] The rule $3AA1$ represents the general logical structure of the rule sentences which regulate how a legal complex sentence becomes valid. According to arts of legal complex sentences such as contract, judgment, administrative order, statute, constitution and convention, the rule sentence $3AA1$ has various forms. The content of the first requirement, «formation» of the complex sentence, the second requirement, «invalidity», the third requirement, «beginning time of effectiveness» or «condition of effectiveness», are to be decided according to the relevant complex sentence. Rule sentences which regulate these requirements are positive law rule sentences which are formed depending on the state or the international law

circumstances.

4.6.1. Positive legal meta-rule sentences regulating the formation of legal sentences ^

[Rz 35] The legal system is composed of legal sentences. Legal rule sentences which regulate the formation of legal sentences are necessarily required for the development of all legal systems. That a sentence is formed as a legal sentence is a precondition in order for that sentence to become legally valid. Legal sentences differ from conventional sentences because a legal sentence must satisfy the requirements of positive legal meta-rule sentences regulating the formation of the legal sentences. Legal complex sentences such as contracts, judgments, statutes, constitutions and conventions are accompanied by positive legal meta-rule sentences which regulate their formation. For example, the following legal rule sentences are positive legal meta-rule sentences which regulate the formation of legal complex sentences (e.g., a contract and a statute).

CISG Article 23: A contract is concluded when an acceptance of an offer become effective¹⁰. Japanese Constitution Article 59: A bill becomes a law on passage by both Houses, except as otherwise provided by the Constitution.

[Rz 36] In our knowledge base, the formation of legal sentence is dealt as one of the requirements of the legal meta-rule sentence to regulate when a legal sentence becomes valid. The first requirement of the rule3AA1, that is introduced above as a legal meta-rule sentence that decides the accrual of validity of a complex sentence, requires that the complex sentence is formed as a legal sentence¹¹. The two positive articles, *CISG Article 23* and *Japanese Constitution Article 59*, are conceivable as sub-rule sentences to regulate the first requirement of variation rule sentences of 3AA1 where «*complex_sentence(CS,ES)*» is replaced by «*contract(CS,ES)*» or by «*statute(CS,ES)*». ¹²

[Rz 37] This rule is to be read: if ES is an elementary sentence of a complex sentence CS and CS is terminated at time T, then ES is terminated at T.

mr4b: is_terminated(obligatory(X,Z,T),T) <-
obligatory(X,Z,T) & perform(X,Z,T).

[Rz 38] This rule is to be read: if it is obligatory for X to do Z at time T and X performs Z at T, then «it is obligatory for X to do Z at time T» is terminated at T.

[Rz 39] In the example, «it is obligatory for Anzai to deliver Bernard the goods», the object sentence is terminated when Anzai delivers Bernard the goods.

4.7. The validity of legal meta-rule sentences ^

[Rz 40] A legal meta-rule sentence which regulates the validity of a legal sentence must be also

proven as valid. How can the validity of legal meta-rule sentences be proven? It depends on the arts of legal meta-rule sentences.

4.7.1. The validity of fundamental legal meta-rule sentences ^

[Rz 41] Fundamental legal meta-rule sentences are generally valid. In other words, the validity of fundamental legal meta-rule sentences is independent of the empirical state of affairs according to the progression of time. Therefore, the proof of the validity of fundamental legal meta-rule sentences is to be done through the assertion of the sentence as true. For example, the validity of the most fundamental legal meta-rule sentence introduced above, $r0$, is asserted as true independent of time. In our legal knowledge base, the following legal fact sentence is set:

$is_valid(r0, T).$

[Rz 42] This formula is to be read: $r0$ is valid at any time. The validity of $r01$, $3AA1$, $3AA2$, $r02$ and $mr4b$ above are to be formalized in the same way.

4.7.2. The validity of positive legal meta-rule sentences ^

[Rz 43] In the case of positive legal meta-rule sentences, their validity is dependent on time and place. The validity needs to be proven through the process of inference by the application of other legal meta-rule sentences to given empirical facts. During this inference process, fundamental legal meta-rule sentences ($r0$, $r01$, $3AA1$, $r02$) as well as positive legal meta-rule sentences are applied. The former is applied so as to decide the validity itself according to the general scheme of the validity. The latter is applied to decide the fulfillment of the requirements of the former. The validity of the former is, as previously described, not dependent on time and place, so that it is to be proven through the assertion that this sentence is valid. The validity of the latter is dependent on time and place, so that it is to be proven again through deduction by the application of further legal meta-rule sentences. In the process of this proof, the fundamental legal meta-rule sentences; $r0$, $r01$, $3AA1$, $r02$; again and further positive legal meta-rule sentences are applied.

4.7.3. The level of the validity of positive legal meta-sentences ^

[Rz 44] In the case that the validity of a positive legal meta-rule sentence is regulated by other positive legal meta-rule sentences, the latter belongs to the higher meta-level in comparison to the former. The latter may be called «higher» legal rule sentence in comparison to the former. The validity of contracts, judgments and administrative orders are regulated by statute law. The validity of statute laws is regulated by constitution. Here one can conceive that contracts, judgments and administrative orders belong to the first level, statutes belong to the higher second level, and constitutions belong to the progressively higher third level of the validity of law¹³.

4.7.4. The basic legal meta-sentences ^

[Rz 45] In state law world, legal meta-rule sentences that regulate the validity of constitutions must be the highest level of legal meta-rule sentences. In the international law world, legal meta-rule sentences that regulate the validity of convention must be the highest level of legal meta-rule sentence. These could be called «basic» legal meta-rule sentence, which is substantially similar to the «Basic Norm» (Grundnorm) by Kelsen¹⁴. The logical structure of basic legal meta-rule sentences which regulate that a constitution as a complex legal sentence becomes valid could be formalized as below:

$$3AA1C: \text{become_valid}(\text{constitution}(CS, E, H), T) \leftarrow$$

$$\text{is_formed}(\text{constitution}(CS, E, H), T1) \quad \& \quad \text{not}(\text{invalid}(\text{constitution}(CS, E, H), T)) \quad \&$$

$$((\text{beginning_time_of_effectiveness}(TE, CS, T) \quad \text{or} \quad$$

$$\text{condition_of_effectiveness}(CE, CS, T)) \text{ or } T=T1).$$

[Rz 46] This formula is to be read: *A constitution CS becomes valid at time T if the constitution CS is formed at time T and it is not the case that the constitution is invalid and (the constitution entails the beginning time of effectiveness and the beginning time has come at T) or (the condition entails a condition of the effectiveness and the condition is fulfilled at time T) or T is T1.*

[Rz 47] This basic legal meta-rule sentence is a variation rule sentence of the fundamental legal meta-rule sentence 3AA1. «*Complex_sentence(CS, ES)*» in 3AA1 is replaced by «*constitution(CS, ES)*» in 3AA1C. 3AA1C provides a general scheme so for a constitution as a legal complex rule sentence to become valid. Each requirement of 3AA1C is to be defined by another legal meta-rule sentence which constitutes the substantial content of the basic legal meta-rule sentences. These are to be presented by a theory of constitution.

[Rz 48] The validity of this basic legal meta-rule cannot be deduced by the application of positive legal rule sentences. The validity must be presupposed so that the validity of constitution can be proven. It must be declared as a legally true fact. The validity is to be declared in our knowledge base as:

$$\text{is_valid}(3AA1C, T).$$

[Rz 49] This formula is to be read: *legal rule sentence 3AA1C is valid at any time.*

[Rz 50] Here it is to be noted that basic legal meta-sentences are not sufficient as a cognitive scheme to systematize law in terms of the validity. Logical Jurisprudence insists that, besides Basic Norm, fundamental legal meta-sentences are to be presupposed to systematize law logically¹⁵.

5. Conclusion ^

- [Rz 51] In this paper, we have at first clarified the logical structure of legal sentences as an element of a legal system in terms of the legal rule sentence and fact sentence, legal elementary sentence and complex sentence, and legal object sentence and meta-sentence. We have then clarified the logical structure of connection of legal sentences to construct a legal system (i.e., connection of an object sentence with a meta-sentence and a legal meta-sentence with another meta-sentence). On the basis of these clarifications, we have tried to clarify the logical structure of system of law as a whole starting from the conception that the system of law is created through legal reasoning.
- [Rz 52] As a result, we have clarified that one's obligation is regulated by the proof of the validity of legal meta-sentence that describes the validity of the legal object sentence describing the obligation. We have clarified how the validity of the legal meta-sentence can be proven through legal meta-reasoning, presenting legal meta-rules which enable such deduction. As legal meta-rule sentences, we have identified the most fundamental meta-rule sentence and other fundamental meta-rule sentences which are implicitly presupposed in legal reasoning. We have identified some positive legal meta-rule sentences which decide the fulfillment of requirements of the fundamental legal meta-rule sentences. We have clarified the ground of the proof of the validity of the fundamental legal meta-rule sentences, presenting relevant legal fact sentence, on the one hand. We have clarified the ground of the proof of the positive legal meta-rule sentences, presenting relevant legal meta-rule sentences including fundamental meta-rule sentences as well as basic legal rule sentences and relevant legal fact sentences, on the other hand.
- [Rz 53] Through the application of the above clarified legal meta-rule sentences, one could constitute the system of valid law. One could justify a decision or deny a decision to a concrete legal problem in terms of the validity of a legal sentence according to the legal system. This paper presents a logical model of a systematization of the validity of law. Whether this model is sufficiently adequate to construct a legal reasoning system of the validity of law requires further demonstrations of the application of these rules to concrete examples. In such demonstrations it is necessary to also show how legal sentences and legal meta-inference work for the proof of the validity of legal sentences in fact. This task will be reserved for our further papers.

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- 1 Cf. *Pufendorf, Samuel*, De jure naturae et gentium liberi actio, Lund (1672), Amsterdam Ed. (1685), English translation from the edition of 1688 by Oldfather, C.H., & Oldfather, W.A., Oceana, New York, p. 208-836 (1964).
 - 2 Cf. *Kelsen, Hans*, Reine Rechtslehre, 2. Auflage, Verlag Franz Deuticke, Wien, S. 196-282 (1960), Kelsen (English translation 1967), *Kelsen, Hans*, (translated by Max Knight from the second German edition), Pure Theory of Law, University of California Press, Berkeley & Los Angeles, p. 193-278 (1967).
 - 3 Cf. *Yoshino, Hajime*, Tractatus Logico-Juridicus – Its Basis, In: *Jakob, R., Philipps, L., Schweighofer, E., & Varga, C. (Hrsg.)*, Auf dem Weg zur Idee der Gerechtigkeit: Gedenkschrift für Ilmar Tammelo, LIT, Wien, S. 127-148 (2009)
 - 4 The representation of legal knowledge in this paper is based on logic programming. Capital letters like «X» or «Y» here are used for variables. For constants are used small letters. Each variable is bound during one (complex) sentence to the end period «.» The implication «<-» is to be interpreted by an inference engine as «:-» in Prolog.
 - 5 CISG is an abbreviation of *The United Nations Convention on Contracts for the International Sale of Goods*.
 - 6 In our knowledge base, this formula is to be represented as: *sen(s2, valid(s1,t04_15_2010))*. «sen» here is a predicate which is abbreviation of «sentence». The first term is a sentence identifier and the second is the content of the sentence. Our inference engine must interpret this data as a sentence identifier and its content in inference.
 - 7 As to systematization of legal meta-inferences for a legal reasoning system, cf. *Yoshino, Hajime*, The Systematization of Legal Meta-inference. In: Proc. The Fifth International Conference on Artificial Intelligence and Law (College Park, MD USA, May 21-24, 1995), ACM (The Association for Computing Machinery), Baltimore MD, p. 266-275 (1995).
 - 8 This meta-rule sentence must be applied to not only the legal reasoning world but also to all reasoning worlds where the validity of knowledge is to be asked. One might say that this rule represents the principle of recognition.
 - 9 The expression «or T is T1» means here: «otherwise the legal complex sentence becomes valid at time when it is formed as a legal complex sentence.»
 - 10 The contract is an art of complex sentences and «conclusion» of the contract means that it is formed as a legal complex sentence named contract. As to our systematization of the CISG Part II regulating formation of the contract see to: *Yoshino, Hajime*, Logical Structure of Contract Law System – For Constructing a Knowledge Base of the United Nations Convention on Contracts for the International Sale of Goods. In: Journal of Advanced Computational Intelligence, Vol.2 No1, Fuji Technology Press, Tokyo, p. 2-11 (1998).
 - 11 It is to be noted that the concept of the formation of legal sentence is not equal to the concept that the sentence becomes legally valid. It is one of requirements of the rule sentence which regulates that the sentence becomes legally valid.
 - 12 As regards to the variation rule sentence of 3AA1, see the rule sentence 3AA/C and the explanation about it in the section 4.74. below.
 - 13 The author agrees with Hans Kelsen who insisted the hierarchical structure of the legal order (den Stufenbau der Rechtsordnung) in this sense. See: Kelsen, FN 2, pp. 228-282 (original) and pp. 221-278 (translation). This paper clarifies, beyond Kelsen, the hierarchical structure as the structure of meta-level relation of legal sentences. This paper also presents logical formalization for grounding of the validity of lower level of law on higher level of law. It enables to demonstrate the grounding as the logical proof of the validity of lower level of legal sentences by the application of higher level of legal meta-sentences.
 - 14 Hans Kelsen proposed the concept of «the Basic Norm» (die Grundnorm). See: Kelsen FN 2, S. 204-209, p. 201-205. This paper formalizes, beyond Kelsen, the logical structure of the basic legal meta-sentence and the grounding the validity of the basic legal meta-sentence logically.
 - 15 It is to be noted that our conception of legal sentence does not always coincide with Kelsen's conception. They differ in the following points: Kelsen starts on legal norms as a meaning, while our theory, Logical Jurisprudence,

starts on legal rule sentences; Kelsen presents Basic Norm (Grundnorm), while we presents not only such a basic legal rule sentence but also the most fundamental legal rule sentence ($r0$) which is always applied to decide the validity of a legal sentence and therefore necessary to construct a legal system of the validity of law. We also identify several fundamental legal meta-rule sentences ($r01$, $3AA1$, $r02$), which regulate that legal sentences become valid or are terminated and the validity of which is to be also presupposed like $asr0$.