This paper analyzes the concept of rights from a logical point of view. We start on two types of legal sentences: legal object sentences and legal meta-sentences. A legal object sentence describes an obligation of its addressee and a legal meta-sentence describes the validity of a legal sentence. We consider a legal sentence one that describes someone’s right as a legal meta-sentence. We clarify logically how legal meta-sentences regulating rights work to establish the validity of legal object sentences in legal reasoning on a concrete legal case.

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1. Introduction

The concept of a right is one of the key concepts of law and theories of law. However, there has been no corresponding definition of the concept of rights. There have been a plethora of controversial definitions for the concept. This variety is partially based on the ambiguity often inherent in the use of the term “rights”. It is necessary to establish a clear definition of a right and formalize the relation of rights and duties in order to both construct total knowledge bases for total reasoning systems of law and develop a genuine science of law.

Scholars have proposed several approaches to clarify the concept of a right in connection to the concept of duty. For example, the Hohfeldian logical formalization approach has been applied to this issue. However, our opinion is that this approach defines the two concepts of rights and duties on the same level of language. This approach fails to adequately systematize the dynamic changes of rights and duties in relationship to changes in time. In contrast, we propose a system of analysis that recognizes the inherent hierarchy between a right on the meta-level and a duty on the object level of language. Legal sentences describing a right work to establish the validity of legal sentences, particularly the validity of legal object sentence. We would like to analyze the concept of right from this point of view according to the approach of Logical Jurisprudence.

This paper takes four steps in the analysis of the concept of a right. First, we clarify the concepts of legal sentences. Legal rule and fact sentences are differentiated from one another as well as from legal object sentences and meta-sentences. Second, we present the fundamental legal meta-rule sentences which regulate the relation between rights and duties. Third, we apply the meta-rule sentences to a concrete problem involving individual rights. This exercise clarifies how a legal object sentence describing an obligation can be deduced by the application of fundamental legal meta-rule sentences and a legal meta-sentence describing a right. Forth, we demonstrate this deduction process through the implementation of a legal meta-inference system to rights and duties.

2. The concepts of legal sentences

Sentences used to express law are our starting points. Our theory refers to these sentences as legal sentences. According to the fundamental concepts of legal sentences, all legal sentences are classified so that law can be systematized. This is necessary to enable the systematization of law as a deductive system.

2.1. Legal rule and fact sentences

First, it is important to distinguish between legal rule and fact sentences. This difference corresponds to “rule” and “fact” in terms of logic programming.
A simple legal rule sentence has this syntactic structure:

\[ a(X) \leftarrow b(X), \, c(X, Y) \].

The consequence of the rule sentence, which is the formula at left in the implication "\( \leftarrow \)", is called a legal consequence and the antecedent, which is the formula at right, is called a legal requirement. Here it is to be noted that the connection between a legal requirement and a legal consequence is expressed merely by means of implication operator "\( \leftarrow \)" in logic programming. To this connection, it is not necessary to give a special philosophical meaning like Kelsen's imputation (Zurechnung).

Legal fact sentences have the following syntactic structure:

\[ b(x_1). \, c(x_1, y_1). \]

It is to be noted that the difference between legal rule and fact sentences is purely syntactic. We do not agree with methodological dualism in terms of norm and fact or "ought" and "is" which Kelsen stands for.

In traditional legal theories, the word "legal norm" is often used. According to our opinion, the object which is to be designated by the word "legal norm" is to be analyzed in terms of the above two types of legal sentences: legal rule and fact sentences. In other words, the concept of a legal norm is to be analyzed not only as legal rule sentences but also as legal fact sentences.

### 2.2. Legal object and meta-sentences

Secondly, it is important for the deductive systematization of legal knowledge to distinguish between legal object and meta-sentences.

A **legal object sentence** describes the object itself. In the legal domain, the object is an "obligation". Legal object sentences prescribe the obligations of a person. The sentence "B must pay A the price of $10,000" is a legal object sentence. In contrast, a **legal meta-sentence** describes the validity of legal sentences. Some legal meta-sentences describe the validity of other legal meta-sentences. An example of a legal meta-rule sentence is:

**CISG Article 99 (1): This Convention enters into force, ... , on the first day of the month following the expiration of twelve months after the date of deposit of the tenth instrument of ratification, acceptance, approval or accession, ... .**

### 3. Fundamental legal meta-sentences
There are fundamental legal meta-rule sentences which implicitly regulate the validity of legal sentences. Through the analysis of CISG and the construction of its knowledge base, we found the following fundamental legal meta-rule sentences which are related to formalizing the relation between rights and duties.

3.1. The most fundamental meta-rule sentence

\[[r0]\] A legal sentence is valid at time $T$, if and only if

the legal sentence becomes valid at time $T_1$ before $T$ &

it is not the case that the sentence becomes null at time $T_2$ before $T$.

This legal meta-rule sentence is not found in any positive law code but implicitly presupposed in every law code. Whenever a legal sentence is applied, this rule sentence is to be applied to decide whether the legal sentence is valid. Therefore, we call this legal meta-rule sentence is the most fundamental of legal meta-rule sentences. Other fundamental legal meta-rule sentences as well as legal meta-rule sentences in positive laws are applied in the process of solving the first and the second requirements of this rule sentence. There are several fundamental legal meta-rule sentences which directly regulate the aforementioned two requirements.

3.2. Fundamental meta-rule sentences regulating the relation between rights and duties

\[[r15]\] There must be the following implicit legal meta-rule sentence which regulates the relation between rights and duties:

\[[3aa2]\] A legal sentence "X has an obligation to do Z at time $T_1$" becomes valid at time $T$, if

Y exercises the right to require X to do Z at time $T_1$ at time $T$ &

the legal sentence "Y has a right to require X to do Z at time $T_1$" is valid at time $T$.

In relation to the concept of right, Kelsen and Hart insist that there is a sort of legal norm which empowers other legal norms. What is the nature of an empowering norm and how is it to be logically formalized? In our opinion, a legal norm being "empowered" by another legal norm means that it can be proven that a legal sentence becomes valid by the application of another legal rule sentences. We think that the latter legal sentence is the meta-legal sentence in comparison to the former legal sentence. In our conception, the legal sentence which becomes valid is not only a rule sentence but also a fact sentence. In contrast, traditional legal
4. Case problem ^

[Rz 17] We introduce a hypothetical case problem 6b, which is simplified from the cases dealt with in our other papers.

[Rz 18] (1) On April 1, a New York manufacturer, A (Anzai), dispatched to the Hamburg branch of a Japanese trading company, gives B (Bernard), a letter containing the following proposal: A will sell B a set of agricultural machines; the price of the machinery is $50,000; A will deliver the machinery to B by May 10; B must pay A the price of the machinery by May 20; the machinery will be transported by an American freight vessel. (2) The proposal reached B's letter box on April 8. (3) On April 9, B telephoned A and said, "I accept your offer".

[Rz 19] (4) A delivered the agricultural machinery to an American freight vessel at the port of New York on May 1. (5) The machinery was delivered to B's Hamburg branch on May 28. B examined the machinery on May 30. (6) B paid A $50,000 on May 31.

[Rz 20] (7) On August 10, the machinery malfunctioned because of a defective gear. (8) B notified A of the malfunction immediately. (9) On September 1, buyer B required seller A to repair the lack of conformity of goods by repair by October 1. (10) A did not repair the defect of the machine by October 1. (11) On October 10, B declared the contract void.

5. Analysis of changes of legal rights and duties relations ^

[Rz 21] We will now analyze changes of legal rights and duties in the case 6b through the application of fundamental legal meta-rule sentences.

5.1. Phases of the change of the validity of legal sentences through the exercise of rights ^

[Rz 22] There are mainly three phases where the exercise of rights affects the validity of legal sentences in case 6b. (1) The contract as a legal complex sentence becomes valid on April 8th through A and B’s exercise of the right to conclude contracts. (2) The legal object sentence “The seller A is obligated to remedy the lack of conformity of the goods by repair” becomes valid on September 1st through the exercise of the buyer B’s right to require the seller A to repair the lack of conformity of the machine. (3) The legal object sentence “buyer B required seller A to repair the goods by October 1” is terminated on October 10 through the exercise of buyer B’s right to declare the contract void. We would like to focus on phase two to further
elaborate how the fundamental legal meta-rule sentences are applied to logically deduce the legal sentences describing such changes of the validity of the relevant legal sentences.

5.2. The occurrence of the obligation to repair the goods

In the case 6b, (7) on August 10, the machinery malfunctioned because of a defective gear and (9) on September 1, B required A that A repairs the lack of conformity by repair by October 1. The CISG admits the right of buyers to require sellers to repair goods in such cases as follows:

Article 46 (3) If the goods do not conform with the contract, the buyer may require the seller to remedy the lack of conformity by repair, ….

However, the CISG does not regulate the obligation of the seller to directly complete the repair. In law, there are hundreds of articles which regulate the occurrence of rights but do not regulate the occurrence of the relevant duties. In such cases, laws presuppose a fundamental legal meta-rule sentence like [3aa2] in order to be able to deduce the relevant obligations.

Seller A’s obligation to make the repairs is deduced through the application of CISG Article 46(3) together with fundamental meta-rule sentence 3aa2 to the facts (7) and (9) of case 6b. The deduction process is as follows:

On the basis of fact (9), the first requirement of the applied rule sentence [3aa2] is proven as:

B required A on September 1 to repair the lack of conformity of goods by repair by October 1.

The second requirement is proven through the application of the fundamental meta-rule sentence as:

„B has the right to A on September 1 to require B to repair the lack of conformity by repair by October 1“ is valid on September 1.

This is proven because the first requirement of the [r0] is proven through the application of CISG article 46 (3) to the fact (7) as:

„B has right to require A to repair the lack of conformity by repair by October 1“ becomes valid on September 1.

The second requirement is proven through the principle of “negation as a failure” because there is no fact that proves that the sentence becomes null.
6. Implementation of the reasoning of the relationship between rights and duties ^

[Rz 29] In order to demonstrate the reasoning to decide the change of the relationship between legal rights and duties on the computer, the relevant legal knowledge is to be mounted as the knowledge base. Furthermore, the inference engine which performs the inference applying the knowledge is to be implemented.

6.1. CPF as a logical representation method of legal knowledge ^

[Rz 30] We have developed Compound Predicate Formula (the abbreviation: CPF) 12 as a logical representation method of legal knowledge. In CPF, a compound predicate term is used additionally to Horn clause logic. It can be represented as follows:

\[ \text{predicate}(ID, \text{CaseList}) \]

[Rz 31] “\text{predicate}” is a predicate name and ID is an identifier of predicate. The identifier is used as a reference to an instance of a predicate. “\text{CaseList}” is a list of case fillers 13. In the case filler, not only a variable or a constant, but also a compound predicate term may appear.

6.2. Representations of fundamental legal meta-rule sentences ^

[Rz 32] The fundamental legal meta-rule sentences above are represented by means of CPF as follows.

\[ [r0]^{14} \text{is\_valid}(IV,[S,T])<- \]

\[ \text{become\_valid}(BV,[S,T1]) \& \text{before}(T1,T) \& \]

\[ \text{not}((\text{become\_null}(BN,[S,T2]),\text{before}(T2,T))) \] 15

\[ [r3aa2] \text{become\_valid}(BV,[\text{obligation}(RE,[X,Y,Z]),T])<- \]

\[ \text{exercise}(EX,[Y,\text{right}(RE,[Y,X,\text{require}(RE,[Y,X,Z,T])]),T]) \& \]

\[ \text{is\_valid}(IV,[\text{right}(RI,[Y,X,\text{require}(RE,[B,A,Z,T])]),T]) \]

[Rz 33] As these fundamental legal meta-rules are assumed to be valid at any time, the following legal fact sentences are set as true:
6.3. Meta-inference engine

The fact (9) in the case 6b, is represented as:

\[
\text{exercise}(\text{ex}, ['B', \text{right}, ['B', 'A', \text{require}(\text{rq}, [\text{buyer}(\text{bu}, ['B']), \text{seller}(\text{se}, ['A']), \text{remedy}(\text{rm}, ['A', \text{the\_lack\_of\_conformity}(\text{lc}, [\text{Goods}, \text{Contract}]), \text{repair}(\text{rp}, ['A', \text{Goods}, \text{by}(t10_01)], \text{by}(t10_01)], t09_01)])], t09_01)).
\]

6.3. Meta-inference engine

The legal meta-inference system with which the changes of rights and duties relations are logically inferred is implemented in the following example. The following is the extracted listing of the legal meta-inference engine written in Prolog:

1. \( \text{solve}(A) : \text{sen}(S, [A]) \).
2. \( \text{solve}(\text{not}(A)) : \text{not}(\text{solve}(A)) \).
3. \( \text{solve}(A \& B) : \text{solve}(A), \text{solve}(B) \).
4. \( \text{solve}(A ; B) : \text{solve}(A); \text{solve}(B) \).
5. \( \text{solve}(A) : \)
6. \( \text{sen}(S, [A <- B]) \),
7. \( \text{solve}(B) \),

\( \ldots \ldots \)

11. \( \text{get\_time\_of\_event}(A, T) \),
12. \( \text{solve}(\text{is\_valid}(l, [S, T])) \).

Line (5) to (12) is the kernel part of the meta-inference. In order to solve goal A, the engine finds a legal rule sentence S, the consequence of which is matched with the goal (6), and solves the requirement B (7). If it is successful, the engine gets the time T of the event from the unified goal A (11) and solves the new goal whether S is valid at the time T (12). If and only if this goal is proven as true, the answer to the original goal is admitted as proven.
6.4. Demonstration of inferring an obligation from the exercise of a right

The following demonstrates how the legal meta-inference works to prove the occurrence of a legal obligation through the exercise of the relevant right applying the above legal meta-rule sentences to the facts of phases (2) in chapters 5.1 and 5.2.

Let us suppose the question in case 6b: what kind of obligation has A to B on September 15? The goal to be solved is by means of CPF represented as:

\[
is\_valid(IV,\{\text{obligation}(OB,\{'A','B','Z'\}),t09\_15)\]

The engine (6) applies rule [r0]. This application matched with the goal, results in engine (7) attempting to solve the unified requirement:

\[
\text{become\_valid}(V,\{\text{obligation}(OB,\{'A','B','Z'\}),T1\}) \& \text{before}(T1,t09\_15) \neg ((\text{become\_null}(BN,[S,T2]),\text{before}(T2, t09\_15)))
\]

The engine (3) tries to solve the following sub-goals separately:

\[
\text{become\_valid}(BV,\{\text{obligation}(Id1,\{'A','B','Z'\}),T1\}) \& \text{before}(T1,t09\_15)
\]

\[
\text{not}((\text{become\_null}(BN,[S,T2]),\text{before}(T2, t09\_15)))
\]

The first sub-goal matches with the rule [r3aa2]. The first unified requirement of rule [r3aa2] is solved on the basis of the fact (9) represented as:

\[
\text{exercise(ex,\{'B','right'\,r,\{'B','A','require'(rq,\{\text{buyer}(bu,\{'B'\}),\text{seller}(se,\{'A'\}),\text{remedy}(rm, \{'A','the\_lack\_of\_conformity'(lc,\{\text{goods,contract}\}),\text{repair}(rp,\{'A','\text{goods}\text{'},by(t10\_01))),by(t10\_01)\},t09\_01)\},t09\_01)).}
\]

The second requirement unified through the resolution of the first requirement is as follows:

\[
is\_valid(IV,\{\text{right}(r,\{'B','A','require'(rq,\{\text{buyer}(bu,\{'B'\}),\text{seller}(se,\{'A'\}),\text{remedy}(rm, \{'A','the\_lack\_of\_conformity'(lc,\{\text{goods,contract}\}),\text{repair}(rp,\{'A','\text{goods}\text{'},by(t10\_01))),by(t10\_01)\},t09\_01)\}),t09\_01)\}
\]

This is to be proven through the application of the most fundamental meta-rule sentence [r0] (further processes of reasoning are not necessary at this point). After the application of [r0] is succeeded, the inference engine (12) solves the goal “r0 is valid on September 1“and the engine (1) is successful because the goal matches with [mf0]. The validity of [r3aa2] is proven in the same manner.
Thus, the answer to the present question is proven as:

\[
is\_valid(IV,\{obligation(Id1,\{'A','B'\}, remedy(RE,\{'A'\}, the\_lack\_of\_conformity(LOC, [Goods,Contract])), repair(REP,\{'A',Goods,by(t10_01)\},by(t10_01))),t09_15\})
\]

This formula is to be read: „A has obligation to B to remedy the lack of conformity by repair by October 1“ is valid on September 15.

7. Conclusion

We have clarified the logical structure of the relation between rights and duties in terms of the legal object sentence and meta-sentence. It has been demonstrated how the validity of a legal object sentence describing one’s obligation is deduced through the application of the fundamental legal meta rule sentences to the relevant facts in the case of the exercise of the other party’s right.

The clarification of the relationship between rights and duties, explains why law maintains social order through the regulation of people’s rights. This clarification makes it possible to formalize the change of the relationship between rights and duties over the course of time as events progress. Thus this clarification contributes to the dynamic systematization of law.

8. Acknowledgements

Thanks to Prof. Jackson Wilson for English assistance.

9. References


1 Hohfeld considered: if X has a right against Y that he shall stay off the former’s land, the correlative (and equivalent) is that Y is under a duty toward X to stay off the place. See: Hohfeld (1913), pp. 32f. Allen and Saxon have tried to formalize this relation logically. See: Allen & Saxon (1993), pp 205-224.


3 This formula is a simplified logic programming expression. This could be read for example: “For all X, X becomes effective, if X is an offer and X reaches the offeree.” This formula could correspond to CISG Article 15(1) which says: “An offer becomes effective when it reaches the offeree.” CISG is an abbreviation of United Nations Convention on Contracts for the International Sale of Goods. “<:-” in the formula is interpreted by our inference engine as “:-” in Prolog (see: chapter 6.3, line 5-7).

4 These formulas could be read for example: “X1 is Anzai’s offer and it reaches Barnard on April 5th”.

5 We agree with Hans Kelsen in that the concept of obligation is essential for law. People try to act in accordance with their legal obligations prescribed by law. In this manner, law helps to control human behavior and realize order in society. We consider obligation is the object of the legal world.
This most fundamental meta-rule sentence was discovered by us using the event calculus approach during the construction of the knowledge bases of CISG. For more information on event calculus, see: Sergot (1986).

Kelsen (2008), pp. 61-63.


It is to be noted that legal object sentences become valid not only by the execution of rights. There are many legal rule sentences directly regulating the obligation of addressees. It is also to be noted that legal meta-sentences describing rights regulate not only the validity of legal object sentences but also the validity of legal meta-sentences.

One may argue that the relevant obligation is also deduced if the „equivalence rule“ between rights and duties like „right(Buyer,Seller,Action)<->duty(Seller,Buyer,Action)“ as in Hohfeld (see: note 1) is applied. However, it is to be noted that the duties of sellers are not always directly deducable from the rights of buyers in law. In many cases in law, rights of one of the parties do not directly result in duties to another party. Law rather presupposes the exercise of rights for occurrence of the relevant obligations. In many case, law leaves the exercise of rights to the choice by the owner of the right. For example, CISG 45 specifies as follows: (1) If the seller fails to perform any of his obligations under the contract or this Convention, the buyer may: (a) exercise the rights provided in articles 46 to 52; (b) claim damages as provided in articles 74 to 77. It is not the case that every relevant obligation of the seller related to each remedy specified in articles 46 to 52 comes out at the time when the seller fails to perform any of his original contractual obligations, but the case that it comes out at the time when the buyer exercises the right selecting it from several rights of remedies. Therefore, we should interpret that law presupposes the fundamental meta-rule sentence like [3aa1].

This is a principle in Logic Programming, It means: the negation of a proposition is to be proven as true when the proof of the proposition is failed. See: the inference line (2) in Chapter 6.3

To CPF see: Yoshino (1997).

The concept of „cases“ here is in principle based on Case grammar (Fillmore, C. J. The Case for Case. In: Bach, Harms (Ed.), Universals in Linguistic Theory, Holt, Rinehart, and Winston, New York, pp.1-88 (1968)). There are Agent case, Object case, Goal case, Location case, Time case and so on. In original CPF, CaseList is a list of pairs which represent case role (symbol) and filler. (Cf. Yoshino & Sakurai (1993), p. 298; Yoshino (1997), pp 82-88.) In this paper, case roles are eliminated for easy representation.

In the knowledge base, the rules are mounted with „sen“ predicate as below: sen(r0,[is_valid(I,[S,T])<-(become_valid(V,[S,T1])&before(T1,T))¬((become_null(N,[S,T2]),before(T2,T1))))].

The validity of legal sentences is related not only to the time but also the place and the matter. For easy understanding, we deal with here only the time case.